The Paradoxes of Quantum Mechanics[1]

The early successes of physics, starting with the work of Galileo, Kepler and Newton, and continuing up to the beginning of the twentieth century, dealt primarily with things that were at least large enough to see and handle. This is the world of our intuition and common sense. Everyone who has learned to play billiards, for example, knows instinctively the Newtonian concepts of force, impulse, momentum, and energy. He or she may not be able to express these ideas mathematically, but in fact the equations only express in a quantitative mathematical way those things that every billiard player knows intuitively and physiologically. The physics of things that are very small, on the other hand, such as atoms, molecules and elementary particles, was developed more recently starting with the work of Niels Bohr, Erwin Schrodinger, Werner Heisenberg and others during the 1920's. We call this body of theory *quantum mechanics*; and by now it has been verified in so many ways that its validity is virtually beyond question. At least as a paradigm for doing precise numerical calculations that can be tested experimentally, quantum mechanics is as accurate and unambiguous as any man made theory is ever likely to be. There is no controversy about this; but when we ask just what this theory is telling us about the ultimate nature of reality, the answers are so strange and counterintuitive that physicists have been arguing about them without any consensus ever since the first principles were formulated almost eighty years ago. Part of the difficulty is that, almost by definition, quantum mechanics deals with things that we cannot see and handle. All the intuition and habits of thought with which evolution has equipped us for surviving in the macroscopic world are utterly useless in this sub-microscopic world. To me at least it seems that there is a further difficulty; it is as if the theory and perhaps reality itself were perversely designed so as to prevent us from pursuing scientific understanding beyond a certain point. With that in mind I will explain some of the concepts of quantum mechanics with special emphasis on those things that seem paradoxical. I will present them in the way that they are usually explained in standard quantum mechanics textbooks, which by and large are not concerned with philosophical questions but only with getting the right answers for certain standard problems. Finally, I will summarize some of the more speculative attempts at making sense of this difficult subject.[2]

---

[2] This discussion assumes no prior knowledge of quantum mechanics. The same material is covered and at much the same level by Alastair I. M. Rae in *Quantum physics: illusion or reality?*, Cambridge University Press, Cambridge, 1986. Those who know some quantum

1 Wave-Particle Duality

Typically, quantum mechanical systems consist of a few particles.[3] The number of particles in ordinary macroscopic things, however, is inconceivably vast. For example, imagine a small pellet of carbon that you could hold between your fingers. If its mass were one mole, fourteen grams, it would contain 6×10^{23} atoms. This huge number is part of the barrier that separates quantum mechanics from everyday reality. Thinking about quantum mechanics requires that we make an imaginative leap over the intervening 23 orders of magnitude into a world in which none of our sense data could ever have any relevance. Let us do that then and think about a single particle of electromagnetic radiation!

Electromagnetic waves are familiar from many contexts. X-rays, ordinary light, and the signals from radio and television broadcasting stations are all examples of electromagnetic radiation. Even though they differ greatly in their properties, they represent the same phenomonon and are described by the same equations. The only real difference among them is their wavelength (or equivalently their frequency), ranging from 500 meters for a typical AM radio signal to 10^{-12} meters in the case of high energy X-rays. Light itself can be produced in many ways. One particularly interesting mechanism involves the electronic structure of individual atoms. An atom can spontaneously change the configuration of its electrons in such a way that it emits light.[4] Light emitted this way from a single atom can be detected by various kinds of laboratory equipment; moreover, the detection always appears to take place *at one instant in time* (to within the time resolution of the detector) and *one point in space* (again, to within the spatial resolution of the detector). This is just what we would expect if the thing that was detected was a particle; so we say that the atom emitted a particle of light called a *photon*, and this photon was detected at one point in space and time by the detector.

Here is our first puzzle; how can something so obviously wave-like as light (or any other form of electromagnetic radiation) also be a particle? The two

---

mechanics might try the survey by Bernard d'Espagnat, *Veiled Reality: An Analysis of Prestent-Day Quantum Mechanical Concepts*, Addison-Wesley, 1995

[3] There are a few odd exceptions to this rule. Superconductivity, for example, manifests itself in macroscopic equipment, but it could not exist in a world without quantum mechanics.

[4] Ordinary fluorescent lights work on this principle. The glass tubes are filled with mercury vapor through which is passed an electrical current. Energy that is absorbed by the atoms from the current is re-emitted by this process as ultraviolet light. This is then absorbed by a coating on the inside of the tube and re-radiated as white light.

attributes seem completely incompatible. For one thing a particle occupies a point in space, whereas a wave must be spread out over a region of space that at least is larger than its wavelength. The standard answer is that light is either wave-like or particle-like depending on what measurements we choose to make on it. For example, if I tune my AM receiver to 550 kHz to listen to my favorite program, I am actually measuring the frequency (or equivalently the wavelength) of the signals coming from the antenna. The receiver is set to amplify those signals with frequencies near 550 kHz and reject all others. Since wavelength is a property of waves (obviously), I conclude that the radio station broadcasts electromagnetic waves. The photon detectors I alluded to in the previous paragraph, on the other hand, measure time and position of arrival, and they can also measure energy. These are properties of particles, and so I conclude that the radiation from individual atoms consists of particles.

But, you are probably wondering, what is light *really* like? What is it like before we make any measurement? Quantum mechanics texts do not answer these questions, and indeed, many physicists regard them as meaningless. At this point you would like to protest; what sort of explanation is it that declares all interesting questions to be meaningless? As a matter of fact, there is some cognitive content in declaring certain questions to be meaningless. We might, for example, try to devise an experiment to measure both wave-like and particle-like properties of light simultaneously. Much ingenuity has been exercised in the quest for such a measurement, but it appears to be impossible. At least, no one has figured out how to do it. The reason that it is (so far as we know) impossible is not because of any shortcomings in our technology, but rather because the laws of physics always intervene in just such a way as to foil any attempt to measure both wave and particle properties together. It is this "conspiracy" that prevents us from even thinking clearly about light, or any other form of radiation, in the absence of measurement. To this extent, the questions really are meaningless.

This explanation comes at a price, however. In classical physics, waves and particles have well-defined properties regardless of whether or not we choose to make measurements on them. If I say that a proton is at a certain position at a certain time with some specified velocity, I mean that there really is a particle out there with these properties. This is part of the notion we all carry with us of an external reality that exists independently of our own thoughts and actions. Somehow the quantum mechanical view of reality cannot be separated from the process of measurement, something that we insititute of our own free will. This is the infamous *measurement problem* in quantum mechanics. I will have more to say about it later.

All forms of electromagnetic radiation travel with the speed of light (at least in vacuum), and according to the theory of relativity, nothing with mass can travel this fast. It follows that photons are massless particles. They carry both momentum and energy, which are simply related, $E = pc$, a formula that is equally valid for photons and electromagnetic waves. (Here E is the total energy of the photon, p is its momentum, and c is the velocity of light.) If the waves have the wavelength λ, then the corresponding photons each carry momentum $p = h/\lambda$, where $h = 6.6 \times 10^{-27}$ erg·sec, a number called Planck's constant. All this was understood, or could have been understood, prior to the development of quantum mechanics. In 1925, however, the French physicist Louis deBroglie made the absolutely crucial speculation that if waves could act like particles, then also massive particles could act like waves. He assumed that the relationship between wavelength and momentum was $p = h/\lambda$, which is now called the deBroglie relation even though it was discovered by Einstein and Planck many years earlier in connection with electromagnetic waves.

There is a clear-cut way to test deBroglie's hypothesis using an experiment called *two-slit interference.* The basic idea is illustrated in Fig. 1. A wave (of any sort) strikes an opaque screen with two parallel slits in it. The waves coming through the two slits overlap one another in the space behind the screen . There will be regions in which the two waves have the same polarity and so reinforce each other. In other regions they will have opposite polarity and cancel out. If the waves consist of light, a screen or photographic film placed behind the slits will reveal a regular pattern of bright and dark areas corresponding to what are called *constructive interference* and *destructive interference.* If one of the slits is covered up, the pattern will disappear and the screen will be uniformly illuminated. The experiment is easy to do with light. Experiments of this sort done in the mid-nineteenth century proved unequivocally that light consists of waves. If one could see an interference pattern with electrons, deBroglie's hypothesis would be proved as well. Unfortunately, the technology required to do this with electrons did not exist in deBroglie's era. The problem is that Planck's constant is so small that the wavelength of any electrons with measurable momentum is many orders of magnitude smaller than the wavelength of light, and the experiment must be correspondingly smaller as well. The experiment has been done recently. It is a brilliant technological tour de force, but of course, the validity of the formula has not been in doubt for many years!

Two-slit interference has a central place in the interpretation of quantum mechanics. The problem is this: suppose that we illuminate the slits with a very weak beam of electrons, let's say one electron per hour or one electron per week. Theory predicts and experiment confirms that the interference pattern develops correspondingly slowly, but the pattern itself does not change. The areas of destructive interference are still there. But what does this one electron interfere with?[5] One might think that the electron splits up and goes through both slits simultaneously so that it interferes with itself, but no one has ever observed an electron split up, and the idea is implausible for other reasons as well. We are forced to conclude something like this: the electron goes *either* through slit A *or* slit B, and these two *possibilities* interfere with each other! But what does it mean to say that possibilities interfere? The guardians of quantum orthodoxy would say that the question is meaningless. Two-slit interference measures wavelength, and so the electrons act like waves. We could try to learn more experimentally by stationing tiny electronic "spies" around each slit to see which slit the electron actually went through. Here again the laws of physics intervene; the very presence of a device that could in principle tell us which slit the particle went through is enough to make the interference pattern disappear! Possibilities interfere, but the detectors turn the possibilities into certainties, which, apparently, don't.

2 The Schrodinger Equation

How do electrons actually behave, and how can we calculate this behavior? In particular, how do they behave in atoms where their wave-like properties should be manifest? The modern formalism for doing such calculations was devised by Erwin Schrodinger. Its centerpiece is the following partial differential equation called the Schrodinger equation.

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial}{\partial t}\psi \tag{1}$$

We will not be concerned with the mathematics, but there are some details that should be pointed out. First, it is an equation for $\psi = \psi(\boldsymbol{r}, t)$, a complex function of space and (sometimes) time called the *wave function.* One solves the equation in order to find ψ. The function itself does not correspond directly to anything that can be measured. For one thing, it has a real and an imaginary part, and no real measurement yields an imaginary number. It is related to measurement as follows: the absolute magnitude

---

[5] The interference of one electron reminds me of the Zen koan, "What is the sound of one hand clapping?"

squared $|\psi(\boldsymbol{x}, t)|^2$ is proportional to the probability of finding the particle at the position $\boldsymbol{x}$ at the time t. Quantum mechanics, as it is usually interpreted, is not deterministic. There is no way to predict where a particle will be at any given time. All that theory can provide is a probability. The only way to check this probability experimentally is to make many measurements on identical systems of particles.

Standard quantum mechanics treats the wave function simply as a tool for calculating certain physical quantities. This avoids all philosophical issues but leaves open the question of what if anything in reality corresponds to the wave function itself. Two slit interference, for example, comes about because the wave function corresponding to the electron passing through slit #1 interacts (at least mathematiclly) with the wave function describing the electron passing through slit #2. The wave function for the complete system (both slits open) is just the sum of these.

$$\psi(\boldsymbol{x}, t) = \psi_1(\boldsymbol{x}, t) + \psi_2(\boldsymbol{x}, t) \tag{2}$$

This simple equation is an example of the *principle of linear superposition,* wave functions corresponding to different possible processes simply add. When we take the absolute magnitude squared to calculate the probability of finding the electron at some point in space, we find that

$$|\psi(\boldsymbol{x}, t)|^2 = |\psi_1(\boldsymbol{x}, t)|^2 + |\psi_2(\boldsymbol{x}, t)|^2 + 2\mathrm{Re}[\psi_1(\boldsymbol{x}, t)\psi_2^*(\boldsymbol{x}, t)] \tag{3}$$

The last term can be positive or negative, so that $|\psi(\boldsymbol{x}, t)|^2$ ranges from zero to $4|\psi_1(\boldsymbol{x}, t)|^2$ depending on whether the interference is completely destructive, completely constructive, or something in between.

Equation (2) is an example of what Schrodinger called an *entangled state.* Somehow the possiblity that the electron went through slit #1, expressed by $\psi_1(\boldsymbol{x}, t)$, is entangled with the possibility that it passed through slit #2, $\psi_2(\boldsymbol{x}, t)$. Thus equation (3) predicts and experiment confirms that there is interference. In that sense the wave function is a reliable tool for performing calculations, but what really is going on, and what does this question mean?

There is a special class of solutions to Schrodinger's equation for which the right side of equation (1) is equal to zero. These solutions do not depend on time so that $\partial\psi/\partial t = 0$. For this reason they are called *stationary states.* One can solve time-dependent problems by simply adding up the stationary states multiplied by time-dependent coefficients. This is another example of the principle of linear superposition. It makes the Schrodinger equation much easier to solve than corresponding classical equations, but it leads to the most notorious of all the puzzles surrounding quantum mechanics, the so-called cat paradox.

3 The Measurement Problem

Imagine (along with Schrodinger who first posed this argument) that there is a cat in a box with some diabolical apparatus. The apparatus contains a single atom of radioactive material. The atom is a quantum mechanical system that will eventually decay into a different element emitting an alpha particle in the process. Because of the non-deterministic nature of quantum mechanics we cannot predict when this will happen, but we can choose the substance so that there is a 50% probability that it will decay within one minute. The apparatus is arranged so that the alpha particle will be detected, and this will cause poisonous gas to be released killing the cat.

Please note that this is a thought experiment only. No one, to the best of my knowledge, has ever done this. In fact there is no point in doing so, because there is no question what one will observe; after one minute the cat will either be alive or dead.

According to the principle of linear superposition, there are two stationary states corresponding to the cat when it is alive and the cat when it is dead. After one minute the wave function is simply

$$\psi = [\psi(\text{live cat}) + \psi(\text{dead cat})]/\sqrt{2}. \tag{4}$$

This is an entangled state, of course, and, as in two slit interference, we can expect trouble. Let's say that the box contains a peephole through which we can look to determine or measure the condition of the cat. If after one minute we look and find the cat is dead, the wave function *collapses* to $\psi = \psi(\text{dead cat})$. The $\psi = \psi(\text{live cat})$ component of the wave function has vanished. By looking in the hole we have killed the cat! If we look and find the cat alive, then the $\psi = \psi(\text{dead cat})$ vanishes, and the cat has a 50% probability of surviving the next minute. In this case we have extended the cat's life by looking in the hole.

We can make this example even more bizarre following an argument suggested by E. Wigner. Suppose that we cannot bear the thought of seeing the cat dead. We therefore go home and arrange for a friend to look in the hole and then call us on the telephone to report the status of the cat. If we regard the friend and the telephone as part of the measurement apparatus, then we kill the cat by listening to his voice on the phone!

No one believes that this argument is exactly right, but there is no consensus on how it should be modified. Most people now believe that the laws of quantum mechanics simply don't apply to macroscopic systems like cats. The reason is that the cat is in intimate contact with its environment. It is constantly bombarded by molecules from the air and the box. There must be some light in the box (otherwise we would not see the cat), so the cat is also bombarded by photons. These interactions in effect are constantly making measurements, for after all, we could in principle use the information contained in the recoil of the particles to infer many things about the state of the cat without ever looking in the peephole. The collisions thus keep the cat wavefunction in a constant state of collapse; which is to say, a cat is a cat, not some strange quantum object.

We have only recently learned to make quantatative models that show how the environment produces the appearance of the classical world. It is likely that Schrodinger understood this intuitively, however, and used the example of a cat only to make his argument more dramatic. The real difficulty is that the cat paradox can be reformulated so that it cannot be explained away by appealing to the environment. To this end consider the apparatus shown in Fig. ? A beam of light hits a half-silvered mirror. According to classical physics, half of the light will pass straight through the mirror and be detected by detector A. The other half will be reflected into detector B. Each detector will register a continuous flux of light with an intensity equal to one-half of the original beam. If we reduce the intensity, however, until it corresponds to just one photon every now and then, and increase the sensitivity of the detectors so that they can detect single photons (this is easy to do with modern technology), then the single photon becomes a quantum mechanical system, and its description is a simple exercise in quantum mechanics. We say that the initial wave function $\psi_\circ$ evolves into a linear superposition of wavefunctions describing the photon headed toward detector A and detector B.

$$\psi_\circ \rightarrow (\psi_A + \psi_B)/\sqrt{2} \tag{5}$$

This is an entangled state similar to the cat that is simultaneously alive and dead. In this case the photon is simultaneously headed toward detector A and detector B. As soon as one of the detectors detects the photon, the wave function instantly collapses into either ψ_A or ψ_B with a probability of 50% each. Thus detecting the photon at A causes the wavefunction at B to vanish. We cannot appeal to the environment to help us out in this case. The photon remains an isolated quantum mechanical system. This setup also shows another paridoxical aspect of the measurement problem. We could in principle let the photon travel arbitrarily far before detecting it. Perhaps we could arrange to have the detectors many light years removed from one another; yet the wave function would still vanish *instantly* at B when the photon was detected at A.

4 Hidden Variables

Einstein and many other physicists have objected to the non-deterministic nature of quantum mechanics. They have argued that the particles must be somewhere at time t, and a theory that cannot predict these positions must be an incomplete theory. Perhaps quantum mechanics is an approximation to some more advanced theory in which the dynamics that govern the exact trajectories of the particles is revealed. In modern terminology we say that such a theory contains *hidden variables,* hidden, that is, from quantum mechanics as we understand it, but possibly revealed in some more complete theory. This is one of the abiding issues in the understanding of quantum mechanics; are there or are there not hidden variables. Modern technology has made it possible to investigate this subject in ways that the founders of quantum mechanics would never have dreamed possible. The results of this investigation are profoundly puzzling. We can understand these results by thinking first about a single electron.

Electrons (as well as most elementary particles) have a property called *spin.* The classical analog of spin is familiar enough; a baseball pitcher makes the ball spin when he wants to throw a curve ball. He snaps his wrist so that the ball rotates about some axis. The direction in which the axis of rotation points and the speed with which the ball rotates around it all depend on how he holds the ball and how hard he throws it. We quantify spin by constructing a quantity called angular momentum. Angular momentum is a vector quantity, that is, it has both a magnitude and a direction. The magnitude is basically a product of the object's mass, size, and speed of rotation. The direction is that of the axis of rotation. Electrons also have angular momentum, but with some surprising differences. For one thing, their size, so far as we know, is really zero. From a classical perspective it would be impossible for an object with zero radius to spin, it just makes no sense, but electrons do. Not only that, every electron in the universe (again, so far as we know) has *exactly* the same spin given by the famous number $\hbar/2$. In the quantum world, $\hbar = h/2\pi = 6.52 \times 10^{-22}$ Mev sec, is the natural unit for angular momentum, and we say that the electron has *spin one-half,* because its angular momentum is half that.

It is possible to measure the direction of an electron's spin by passing it through a specially constructed magnetic field. The electron is then deflected either parallel or antiparallel to the field. By measuring the deflection we can determine the component of spin in the direction of the field. This is called a *Stern-Gerlach experiment* after the physicists who invented the technique. The peculiar thing is that whenever we perform this experiment on an electron the result is always either $\hbar/2$ or $-\hbar/2$. In other words, the spin is always pointing parallel or antiparallel to the field no matter what direction we orient the field! This result is predicted by quantum mechanics, so in this sense it is not surprising. According to the standard interpretation the electron's spin direction is not defined prior to the measurement. If we ask what is the direction before the measurement, we are simply asking a meaningless question. When the electron interacts with the measuring apparatus, however, it spontaneously and unpredictably orients itself parallel or antiparallel to the field. This is another case of entangled wave functions. Before the measurement the electron's wave function can be written as a linear superposition of a spin-up state and a spin-down state (with respect to any axis).

$$\psi = [\psi(\text{spin up}) + \psi(\text{spin down})]/\sqrt{2}. \tag{6}$$

After the measurement the wave function collapses to pure spin up or spin down.

This situation is even more paradoxical when there are two electrons in the picture. Let's suppose that some quantum system emits two electrons, which are ejected in opposite directions. The nature of the system is such that the total angular momentum of the two is zero. Since all electrons have the same spin, it seems reasonable that the two spin axes must be pointing in opposite directions; and indeed, if we measure the spins with two Stern-Gerlach magnets arranged in such a way that their fields both point in the same direction, then one electron will *always* be deflected up and the other down. According to the standard interpretation, however, each electron spontaneously and unpredictably orients itself parallel or antiparallel to the field. So if one electron spontaneously and unpredictably orients itself with its spin up, how does the other one know to orient itself spontaneously and unpredictably with its spin down?

This experiment with the two electrons should be regarded as a thought experiment;[6] quantum mechanics predicts unambiguously what the results will be, but the experiment is impracticable to do for technical reasons. Equivalent experiments have been performed with photons with the detectors several miles apart.[7] The photons are always detected with opposite spin directions. To put this in human terms, it is as if two people part

---

[6] This experiment seems to have been suggested first by David Bohm in *Quantum Theory*, Prentice-Hall, Englewood Cliffs, N. J. (1951) pp. 614-619. An earlier version appears in a famous paper by A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47**, 777 (1935).

[7] The electron experiment is easier to explain, however.

company and travel for miles in opposite directions. Eventually both travelers come to forks in the road. Each traveler decides spontaneously and unpredictably to turn right or left, but it happens that they make opposite choices. The experiment is repeated many times. The travelers always take opposite turns, despite the fact there is no apparent way they could communicate with one another.

The story with the human travelers could be explained if they had some prior understanding when they parted. One person could say something like, "If you come to a fork in the road, take a left turn. I'll take a right." Perhaps the paradox of the two electrons could be explained in the same way; there is some prior understanding (whatever this may mean) between them on the general subject of how they should behave when encountering a Stern-Gerlach magnet. This is a figurative way of stating the content of the local hidden variable hypothesis, *i.e.* there is some level of reality inaccessible to ordinary quantum mechanics that determines which way the electrons are deflected. (I will explain the significance of the qualifier *local* presently.) The human analogy also hints at a way to test the hypothesis. Suppose that, rather than encountering forks, the travelers came upon complicated intersections that were designed to baffle any amount of prior planning. This is the content of a remarkable experiment first proposed by John Bell[8] years ago and recently performed with many variations.[9] I will describe the Bell experiment in a version published by David Mermin.[10] It is easier to explain than the original, but impracticable because it involves electrons. The results of the original Bell experiment (done with photons) as well as the rules of quantum mechanics leave little doubt what the results would be if the experiment could be performed.

The experiment is pictured schematically in Fig. ? Some device emits two electrons in opposite directions. They are in a state of zero total angular momentum. Each electron encounters a Stern-Gerlach magnet with its axis perpendicular to the electron's direction. If the axes of these two magnets point in the same direction, the experiment is identical to the one described above. If one electron is deflected down, the other will always be deflected up. In the Mermin version of the experiment there are two other settings possible for the magnets' directions. Each independently can be turned 120 degrees to the right or left of straight up. Thus each magnet

---

[8] J. S. Bell, Physics *1*, 195 (1964).

[9] A. Aspect, P Grangier, G. Roger, Phys. Rev. Lett. *47*, 460 (1981). A. Aspect, P Grangier, G. Roger, Phys. Rev. Lett. *49*, 91 (1982). A. Aspect, J. Dalibard, P Grangier, G. Roger, Phys. Rev. Lett. *49*, 1804 (1982)

[10] N. D. Mermin, Physics Today, April 1985, pp 39-47.

has three settings that we can call 1, 2, and 3. (It makes no difference how we number them so long as both magnets are numbered in the same way.) If both magnets have the same setting, whether it be 1, 2, or 3, the results of the experiment are unambiguous. If we look at any one detector, the electrons will be deflected at random up or down. If we look at both detectors simultaneously, however; one electron will always be deflected up (with respect to the magnet's axis) and the other will be deflected down. If the setting are different, it is possible for both electrons to be deflected up (with respect to different directions, of course) or down, and we can ask about the relative probabilities of these different kinds of events.

To measure these probabilities we will repeat the experiment many times, each time choosing in some random and unpredictable way the two detector settings. Our data will consist of the detector settings and the results of the two spin measurements. When we examine this data we will find that the pattern of deflection is random, but in the limit of many events it turns out that exactly half the time the electrons are deflected in the same direction and half the time in opposite directions.

Suppose that the hidden variable hypothesis is correct. We can picture this by imagining that each electron carries with it a detailed set of instructions that tells it what to do under all conceivable circumstances. One of these circumstances is the experiment I have just described with the three magnet settings. The instructions might say something like, "If you come to the magnet with the three settings and it is currently set to position 2, go into the up position." The other electron, of course, has its own set of instructions; the two instruction sets were issued jointly when the electrons started out. Here in an obvious notation is a typical set for the two electrons:

$$(1U, 2D, 3D)(1D, 2U, 3U). \tag{7}$$

If both detector settings are the same, the electrons will be deflected in opposite directions. If on the other hand the setting are 1 and 2 in this example, both electrons will be deflected up.

It is important to remember that the entity that is making up the instruction sets has no idea what the detector settings will be. This is a metaphoric way of explaining what is meant by a local hidden variable theory. (There are also *global* theories in which the the particles, the reaction that produces them, and perhaps even the detectors have some way of communicating with one another.) The instructions must be drawn up in such a way, however, that *if* the settings happen to be identical, the particles will be deflected in opposite directions. This is very limiting. There are instructions like (7) above with two D's and a U for one particle and two U's and a D for the other, and instructions like

$$(1U, 2U, 3U)(1D, 2D, 3D) \tag{8}$$

with three U's for one particle and three D's for the other. Nothing else will work. In the first instance there will be five combinations that produce opposite deflections, for example 11, 22, 23, 32, and 33 in (7), and only four combinations that produce the same deflection. In the second instance, for example (8), all nine settings produce opposite deflection.

That's all there is to it. Experimentally we find that there are an equal number of events with the same and opposite deflections. This result is trivial to calculate with quantum mechanics, but impossible to explain on the basis of some metaphorical set of instructions carried by the particles. This is the content of what has come to be called *Bell's theorem*; quantum mechanics is inconsistent with the local hidden variables hypothesis. We know from experiments that quantum mechanics is right. Therefore there can be no local hidden variables.

The quantum mechanical result would not be so difficult to understand if the particles could somehow communicate with one another. Our human travelers could carry cellular phones. When they got to the forks in the road they could call one another and discuss some plan of action. Does this analogy help explain the quantum mechanics?

According to the theory of relativity, it is not possible to send information faster than the speed of light. No one has been able to discover a loophole in this law.[11] According to quantum mechanics, however, the electrons make their decisions *instantaneously.* To some extent, the modern versions of Bell's experiment have been able to test this. They show that *if* the electrons communicate, they must do so at supraluminary speeds.

So if the electrons, in effect, communicate with one another instantaneously, why can't we use some version of Bell's experiment to send messages faster than light in violation of the theory of relativity? The answer is that we cannot see any effect of their communication until after we have looked at the data from *both* detectors. If we just look at the data from one detector, we will see that the electrons are deflected up and down randomly with equal probability, like the results of tossing a coin, regardless of what is being done at the other detector. It is only when we compare the data from both detectors that we see these strange correlations that suggest that each

---

[11] Velocities faster than light called *phase velocities* do appear in physics. They cannot, however, carry information.

electron “knows” just what the other is doing. So if electrons do communicate with one another instantaneously, they do so in a way that we have no access to and can never use for our own purposes.

5 Quantum Mechanics and Philosophy

The puzzles or paradoxes reviewed above all involve entangled states. They are puzzling because the wave function to which we have no direct experimental access seems to describe things that cannot happen in the real world as we understand it; information is propagated with infinite velocity, wave functions instantaneously collapse because of measurements made in distant regions of space, events that might not have happened interfere with one another, *etc.* I must emphasize that one can *do* quantum mechanics without ever thinking about these problems, but since this is a book about physics and philosophy, it is imperative that we address the rich philophical content in these examples. To this end it is useful to review some ideas from traditional philosophy.

We usually think of science as being objective in the dictionary sense of pertaining to material objects as distinguished from mental concepts, ideas, or beliefs. Many philosophers, on the other hand, have regarded the existence of external material objects as a dubious concept. This unease stems from the fact that we have no way of knowing the world apart from our sense data and our memories of this data, and we have no independent way of checking this knowledge that does not itself rely on the same data and memory. I could argue, for example, that nothing exists except my own mind, which came into existence a few minutes ago. The real world is just an illusion, and my memory of the past is an illusion also. This doesn’t seem like a very useful proposition, but I can think of no way of refuting it based on reason alone. Some philosophers have based their views on hypotheses that are almost as extreme. The English philosopher George Berekeley (1685-1753) maintained, for example, that nothing exists except minds and percepts.[12] This school of thought that denies the primacy of objective reality is usually called ‘idealism.’ Berkeley’s position is so extreme in this regard that it might be useful to call it ’radical idealism,’ but there are less extreme versions to be found among the philosophers of his era including Locke, Hume, and Hegel.

The radical idealists provide little help or motivation for doing science.

---

[12] He also included God and spirits in his list of things that exist, which seems somewhat inconsistent.

After all, if the objective world is just an illusion, why bother to study it? None of these philosophers seem to have noticed that the world *seems* to be full of peolple who *seem* to view the world much as we do and *seem* to recall past events that are consistent with our memories as well. This intersubjective agreement does not exactly disprove the idealists' claims, these other people might all be part of the illusion, but by the same token, it makes it difficult to take them very seriously. There is one idealist philosopher, however, whose work at least partly avoids this criticism and provides some useful insights for physics as well. That is Emmanuel Kant (1724-1804).

Acording to Kant, the objective world provides the raw material of our sense data, but our own mental apparatus supplies the concepts and structures by means of which we understand this data, and in particular, it supplies the concepts of space, time, and causality, which do not exist apart from our own minds. The elements of external reality, Kant calls them "things in themselves" or *noumena*, are basically unknowable; they are not in space or time, and they cannot be described by the Kantian 'categories,' such as unity and plurality, possibility and necessity, accident and cause-and-effect. These are held to be a priori concepts; they are not learned by observing the objective world (although such observations may elicit them), rather they are part of the basic apparatus of our minds. This is a promising argument to account for the intersubjective agreement that we observe. Presumably we can all agree that there is a teapot on the table, because we are all perceiving the same noumena using mental apparatus that is common to us all.

This is not the place to review Kant's arguments, which are likely to appear far-fetched to the modern reader.[13] The point is that the quantum world may be every bit as shadowy and unknowable as Kant believes all of reality is. Certainly it is difficult to reconcile collapsing wave functions, faster than light communication, and delayed choice experiments with our notions of space, time, and causality. For this reason some physicists welcome the weirdness of quantum mechanics as a window, albeit a very cloudy one, into a level of reality that lies outside of space and time and perhaps underlies macroscopic reality, which we succeed in ordering according to our categories of space, time, locality, and causality. Bernard d'Espagnat[14] has coined the term *veiled reality* for this deeper level of reality and contrasts it with *impirical reality* as it is revelaed by our senses, our intuition, and our

---

[13] A good review is to be found in *A History of Western Philosophy* by Bertrand Russell, Simon and Schuster, (1945)

[14] *Veiled Reality*, B. d'Espagnat, Addison Wesley, 19??

measuring apparatus.

There is another school of thought, however, that holds that we *could* if we had a complete theory, understand all of quantum mechanics in the language of classical physics. In this view particles have definite positions and trajectories, and cause and effect unfolds in a way that is consistent with the theory of special relativity. Such interpretations are called collectively, *local realism.* In general, quantum mechanics makes a good fit with Kantian idealism. There is very little that needs to be reinterpreted. Realists, on the other hand, have some explaining to do. We review here some modern attempts to reconcile quantum mechanics with realism.

5.1 The Pilot Wave Theory

Another name for quantum mechanics is *wave mechanics,* and the $\psi(\boldsymbol{x}, t)$ that appears in Schrodinger's equation is the wave function. In fact, Schrodinger's equation looks like a deterministic equation for classical wave motion. The root of all our problems is that we have no idea what is waving. When we make an observation in the quantum world, we find not waves but particles. This is one way to look at the paradox, how can one have a wave theory of particles?

There is an elegant solution to this problem first proposed by Louis de-Broglie in his pilot wave theory of 1927.[15] It was later rediscovered by David Bohm[16] and hence sometimes called *Bohmian mechanics.* It has not found much acceptance among physicists and is seldom mentioned in textbooks, but it has been championed by such latter-day gurus of the foundations of quantum mechanics as John Bell,[17] B. d'Espagnat[18], and Sheldon Goldstein.[19] de Broglie's idea is that quantum mechanics is a theory about waves *and* particles. The waves are described by Schrodinger's equation exactly as in conventional quantum mechanics. There is a new equation, however, that describes the motion of particles. In the simple case where there is only one particle involved it is

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{v}(\psi, \boldsymbol{X}) \tag{9}$$

---

[15] L. de Broglie, in *Rapport au V'ieme Congres de Physique Solvay.* Gauthier-Villars, Paris (1930).

[16] D. Bohm, *Phys. Rev.* **85**, 165, 180 (1952).

[17] *Speakable and unspeakable in quantum mechanics.* Cambridge University Press, Cambridge, U.K. (1993)

[18] *Veiled Reality: An Analysis of Prestent-Day Quantum Mechanical Concepts*, Addison-Wesley, 1995

[19] *Physics Today*, Vol 51, No. 4 (1998)

Here $\boldsymbol{X}$ is the position of the particle at time t, and $\boldsymbol{v}$ is a velocity calculated in a straightforward way from a knowledge of the wave function ψ. It is also classical and deterministic in the sense that, given the starting coordinates of the particles, the wave function determines their exact subsequent paths. The wave function just shepherds them around in such a way that in regions where $|\psi(\boldsymbol{x}, t)|^2$ is large there is a correspondingly greater probability of finding the particles than where $|\psi(\boldsymbol{x}, t)|^2$ is small. In this view there is no measurement problem and no collapse of wave functions. What one detects in an experiment is quite simply the positions of particles. We cannot predict the outcome of such measurements only because, in general, we don't know the starting coordinates well enough to do the calculation implied by (9). This is just like thermodynamics where we can calculate the pressure and velocity of a gas without ever being able to calculate the trajectories of the individual gas molecules.

The pilot wave theory does have two shortcomings that have perhaps prevented it from being accepted as the correct explanation of quantum mechanics. The first problem has to do with the meaning of ψ. It is sometimes called the *Bohm-de Broglie field,* but in fact we still don't know what is waving. There is no way to explain its existence, and no way to prove or disprove the theory experimentally. It is guaranteed to give the same results as conventional quantum mechanics. The second problem has to do with relativity. The pilot wave theory is non-relativistic, and it has not been possible to make a relativistic version of it. This is not surprising, of course; quantum mechanics does apparently propagate some kinds of information at infinite velocity in clearcut violation of relativity. And yet — relativity is clearly right about so many things we would like to have some insight about how quantum mechanics and relativity coexist. The pilot wave theory cannot do this.

Perhaps the most valuable service performed by the pilot wave theory has been to serve as a theoretical laboratory where the consequence of general theorems about quantum mechanics can be worked out in detail. For example, there is a well-known theorem by John von Neumann[20] to the effect that hidden variables are inconsistent with quantum mechanics: there can be no quantum mechanical theory with hidden variables. The de Broglie-Bohm model is just such a theory of course, and the von Neumann theorem is wrong.[21]

---

[20] J. Von Neumann: *Mathematische Grundlagen der Quantenmechanik*, Berlin (1932) (English translation (Princeton, 1955))

[21] See for example, J. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge (1993), especially Chapter 4

5.2 The Many Worlds Interpretation

There is a fantastic version of quantum mechanics published by H. Everett in 1957 and occasionally supported by other physicists.[22] You will recall that in Schrodinger's thought experiment, the cat is in an entangled state, simultaneously alive and dead, until some measurement causes the wave function to collapse into either pure live-cat or dead-cat states. In the Everett theory there is no collapse, rather the universe splits! In one universe the cat is alive and in the other the cat is dead. In other respects the universes are identical containing, for example, identical copies of the apparatus and the observer. In one, however, the observer retains the memory of the live cat and in the other, the observer remembers the dead cat. Everett is able to show with a simple model that neither observer is aware of the split.

How often does this happen? According to Everett, the universe splits every time a *measurement like* process ocurrs. He never explains exactly what this means, but perhaps every interaction between systems that could in principle determine the quantum state of one of the systems is enough to cause the universe to split. By this reckoning there must be an inconceivably vast number of universes existing simultaneously with more or less identical copies of dead cats, meddling physicists, and galactic superclusters. A truly extravagant theory!

Except for its shock value, the Everett theory doesn't have much to offer. There is no way to test it experimentally, since it is guaranteed to be consistent with whatever we observe (in this universe). It is also guaranteed to be consistent with conventional quantum mechanics except for its explanation of the measurement problem.

5.3 The Consistent Histories Approach

We conclude with a relatively new idea that has been the subject of active research in recent years. It is called the *consistent histories* or *decoherent* histories approach, and it was proposed independently (in different versions) by Robert Griffiths,[23] Roland Omnes,[24] and Murray Gell-Mann and James

---

[22] H. Everett, *Reviews of Modern Physics* **29**, 454 (1957); see also B. S. de Witt *Physics Today* **23** (1970), No. 9, p. 30

[23] R. Griffiths, J. Stat. Phys. **36**, 219,(1984).

[24] R. Omnes, J. Stat. Phys. **53**, 893, 933, 957 (1988).

Hartle.[25][26] The object of these formulations is to avoid all reference to quantum measurement and thus to avoid the collapsing wave functions that are such an embarassment to conventional quantum mechanics. To this end quantum processes are described in terms of histories. Let's return to the two-slit interference experiment, Fig. 1, for a simple example.

A particle leaves the source at a and passes through slit b or c. We describe these two possibilities with the two histories

$$\psi_a \rightarrow \psi_b \tag{10}$$

$$\psi_a \rightarrow \psi_c$$

These symbols should be interpreted in the obvious macroscopic way, "the particle from a goes through slit b" or "the particle from a goes through slit c." The usual rules of quantum mechanics assign a probability of 50% to each of these histories. According to this approach, quantum mechanics is fundamentally a theory about probabilities. We can, however, write the deterministic history

$$\psi_a \rightarrow (\psi_b + \psi_c)/\sqrt{2} \tag{11}$$

Both (10) and (11) are valid or consistent histories.

Now single out two spots on the screen and call them d and e. In the region of d there is complete destructive interference and the screen is dark. In the nearby region e there is complete constructive interference. We might suppose that a particle reaches e by means of one of the following two histories

$$\psi_a \rightarrow \psi_b \rightarrow \psi_e \tag{12}$$

$$\psi_a \rightarrow \psi_c \rightarrow \psi_e$$

Part of the mathematical formalism of these theories is an algorithm for deciding which histories are consistent. Neither of the histories in (12) is. The following two histories, however, are consistent.

$$\psi_a \rightarrow \psi_b \rightarrow (\psi_d + \psi_e)/\sqrt{2} \tag{13}$$

$$\psi_a \rightarrow \psi_c \rightarrow (-\psi_d + \psi_e)/\sqrt{2}$$

---

[25] M. Gell-Mann, J. B. Hartle, in *Complexity, Entropy, and the Physics of Information*, W. Zurek, ed., Addison-Wesley, Reading, Mass. (1990), p. 425. M. Gell-Mann, J. B. Hartle, Phys. Rev. D **47**, 3345 (1993).

[26] For a less technical introduction see S. Goldstein, *Quantum Theory Without Observers — Part One* Physics Today, **51**, No. 3 (1998) p.42 and R. B. Griffiths and R. Omnes, *Consistent Histories and Quantum Measurements* Physics Today, August (1999), p. 26

Combining (11) and (13) then yields

$$\psi_a \to (\psi_b + \psi_c)/\sqrt{2} \to \psi_e \tag{14}$$

This is consistent and corresponds to what we actually see, the particle winds up at the spot e. But did it go through b or c? We cannot logically ask this question, because both answers correspond to inconsistent histories.

The formalism so far has reproduced the usual quantum mechanical results without invoking quantum measurements, observers, or collapsing wave functions. It seems too simple, however, and in a subtle way it is. The problem is that the usual criterion for deciding which histories are consistent is too generous. It allows histories that are inconsistent with Bell's theorem and various other hidden-variable theorems as well. For this reason the consistent histories approach is still an area of active research. Its practitioners believe that a set of criteria for consistency will eventually be developed that will also reproduce all the usual rules of quantum mechanics. It remains to be seen whether this final formulation will be sufficiently simple and elegant to allay our misgivings about quantum measurement theory.