These problems are due Friday, March 14.

1. In class I defined the response function

\[ G(\tau) = \left( \frac{2e^2 N}{m} \right) \int_{-\infty}^{\infty} \frac{d\omega}{\omega_0^2 - \omega^2 - i\omega \gamma} e^{-i\omega \tau} \]

Do the integral and plot your results. Verify that it’s real and causal.

2. A stylized model of the ionosphere is a medium described by the plasma function

\[ \epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \]

where \( \omega_p \) is the plasma frequency. Consider the earth with such a medium beginning suddenly at a height \( h \) and extending to infinity. For waves with polarization perpendicular to the plane of incidence (from a horizontal antenna) show from Fresnel’s equations for reflection and refraction that for \( \omega > \omega_p \) there is a range of angles of incidence for which reflection is not total, but for larger angles there is total reflection back toward the earth.

3. An approximately monochromatic plane wave packet in one dimension has the instantaneous form, \( u(x,0) = f(x)e^{ik_0x} \), with \( f(x) \) the modulation envelope given by

\[ f(x) = Ne^{-\alpha|x|/2} \]

Calculate the wave-number spectrum \( |A(k)|^2 \) of the packet, sketch \( |u(x,0)|^2 \) and \( |A(x)|^2 \), evaluate explicitly the rms deviation from the means \( \Delta x \) and \( \Delta k \) (defined in terms of the intensities \( |u(x,0)|^2 \) and \( |A(k)|^2 \)). Verify the “uncertainty relation,”

\[ \Delta x \Delta k \geq \frac{1}{2} \]