These problems are due Friday, February 22.

1. An ideal circular parallel plate capacitor of radius \( a \) and plate separation \( d \ll a \) is connected to a current source by axial leads. The current in the wire is \( I(t) = I_0 \cos \omega t \). (Jackson problem 6.14)

   (a) Calculate the electric and magnetic fields between the plates to second order in powers of the frequency (or wave number), neglecting the effects of fringing fields.

   (b) Calculate the volume integrals \( w_e \) and \( w_m \) that enter in the definition of the reactance \( X \) to second order in \( \omega \). As we showed in class

   \[
   X = \frac{4\omega}{|I_i|^2} \int_V (w_m - w_e) \, d^3x
   \]

   \[
   w_e = \frac{1}{4}(\mathbf{E} \cdot \mathbf{D}^*) \quad w_m = \frac{1}{4}(\mathbf{B} \cdot \mathbf{H}^*)
   \]

   Show that

   \[
   \int w_e d^3x = \frac{|I_i|^2 d}{\omega^2 a^2} \quad \int w_m d^3x = \frac{|I_i|^2 d}{8c^2} \left( 1 + \frac{\omega^2 a^2}{12c^2} \right)
   \]

   Here \( I_i = -i\omega Q \), where \( Q \) is the total charge on one plate.

2. Consider the Dirac expression

   \[
   \mathbf{A}(\mathbf{R}) = -g \int_{-\infty}^{0} d\ell' \times \nabla \left( 1/|\mathbf{r} - \mathbf{r}'| \right)
   \]

   for the vector potential of a magnetic monopole and its associated string as discussed in class. Suppose as we did that the monopole is located at the origin and the string along the negative \( z \) axis. (Jackson 6.18)
(a) Verify my claim that

\[
\mathbf{A} = \frac{q \hat{\phi}}{r} \left( \frac{1 - \cos \theta}{\sin \theta} \right)
\]

(b) Verify that \( \mathbf{B} = \nabla \times \mathbf{A} \) is the Coulomb-like field of a point charge, except perhaps at \( \theta = \pi \).

(c) With the \( \mathbf{B} \) determined in part b, evaluate the total magnetic flux passing through the circular loop of radius \( R \sin \theta \) centered on the \( z \) axis. Consider \( \theta < \pi/2 \) and \( \theta > \pi/2 \) separately, but always calculate the upward flux.

(d) From \( \oint \mathbf{A} \cdot d\mathbf{l} \) around the loop, determine the total magnetic flux through the loop. Compare the result with that found in part c. Show that they are equal for \( 0 < \theta < \pi/2 \), but have a constant difference from \( \pi/2 < \theta < \pi \). Interpret this difference.