Problem Set #1
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These problems are due Friday, January 18.

1. In class we derived the following definition for the magnetic moment of a flat loop of wire.
   \[ m = \hat{n}I \times \text{area of loop} \]
   This can easily be generalized to a loop that is not flat.
   \[ m = I \int_{s} \hat{n} \, da \]
   When discussing vector potentials we came up with
   \[ m = \frac{I}{2} \oint r \times dr \]
   Prove that these are equivalent.
   Hint: There are probably many ways of doing this, but I found the following (obscure) variant of Stokes’s theorem useful
   \[ \oint dr \times A = \int_{S} (\hat{n} \times \nabla) \times A \, dS \]

2. A sphere is surface charged uniformly with charge density \( \sigma \) and rotated with angular velocity \( \omega \) around the polar axis. Find \( B \) and \( H \) everywhere.
   Hint: Since the magnetic scalar potential \( \phi_{M} \) satisfies \( \nabla^{2} \phi_{M} = 0 \) you can expand \( \phi_{M} \) in Legendre polynomials just as you would in an electrostatics problem. Then use the boundary conditions on \( B \) and \( H \) to fix the terms on the expansion.

3. Franklin 8.3.