1. Answer the following questions with a few well-chosen words, equations or diagrams.

(a) According to Dirac, the magnetic monopole charge $g$ should be quantized according to
$$\frac{eg}{\hbar c} = \frac{n}{2}$$
Explain how this comes about.

(b) Suppose a sphere with constant, known permeability is placed in an initially uniform magnetic field. How would you go about finding the resulting magnetic fields. You don’t have to calculate anything. Just show how you would set up the problem.

(c) So long as there are no magnetic monopoles, $\nabla \cdot \mathbf{B} = 0$. Under what circumstances is $\nabla \cdot \mathbf{H}$ not equal to zero? What is it equal to?

(d) Under what circumstances are $\mathbf{E}$ and $\mathbf{B}$ guaranteed to be perpendicular to the direction of propagation of an electromagnetic wave? When might they not be?

2. An ideal circular parallel-plate capacitor of radius $a$ and plate separation $d << a$ is initially charged with total charge $Q_0$. At $t = 0$ it is discharged through a resistance $R$ so that the charge on the plates decays according to the usual rule:
$$Q(t) = Q_0e^{-t/RC}$$
Assume that the decay takes place so slowly that the plates are uniformly charged at all times. In order to make this easier to grade let’s all assume that the positively charged plate is on top ($+z$ direction) so that $\mathbf{E}$ points in the $-z$ direction.
(a) Calculate the electric field between the plates as a function of time. Express your result in terms of $Q(t)$. (Neglect the electric field induced by the changing magnetic field.)

(b) Calculate the magnetic field between the plates resulting from the changing electric field. Express your answer in terms of $\dot{Q}$.

(c) Assume that the leads are connected axially as shown in the sketch so that the current flows radially inward or outward to or from the center of the two plates. Calculate the surface current $K(\rho)$ as a function of $\rho$. Express your result in terms of $\dot{Q}$.

(d) Calculate the magnetic field between the plates resulting from the surface current in the plates. Be very careful with your signs.

(e) Calculate the Poynting vector between the plates. Again, express your result in terms of $Q$ and $\dot{Q}$.

(f) The result for the Poynting vector looks to me rather strange. How do you understand its physical significance? Please explain.

(10 points for each part)

Your will need some of the following formulas:

$$\nabla \cdot A = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times A = \hat{r} \left( \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + \hat{z} \frac{1}{\rho} \left( \frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{\partial A_\phi}{\partial \phi} \right)$$
\[ \phi = \sum_l \left[ A_l r^l + \frac{B_l}{r^{l+1}} \right] P_l(\cos \theta) \]

\[ \int_S (\nabla \times A) \cdot \hat{n} \, da = \oint A \cdot dl \quad \text{Stokes' law} \]

\[ B = \nabla \times A \quad B = \mu H \quad H = B - 4\pi M \quad H = -\nabla \phi_M \]

\[ \sigma_M = M \cdot \hat{n} \quad \rho_M = -\nabla \cdot M \]

\[ \phi_M = \int \frac{\sigma_M \, da}{R} + \int \frac{(\rho_T + \rho_M)}{R} \, dV \]

\[ \hat{n}_{21} \times (H_2 - H_1) = \frac{4\pi}{c} K_T \quad \hat{n}_{21} \cdot (B_2 - B_1) = 0 \quad \phi_1 = \phi_2 \text{ if } K_T = 0 \]

\[ J_M = c \nabla \times M \quad K_M = cM \times \hat{n} \]

\[ \nabla \cdot E = 4\pi \rho \quad \nabla \cdot B = 0 \quad \varepsilon = -\frac{1}{c} \frac{\partial \Phi}{\partial t} \]

\[ \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t} \quad \nabla \times H = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial D}{\partial t} \]

\[ E = -\nabla \phi - \frac{\partial A}{\partial t} \quad B = \nabla \times A \]

\[ \frac{\partial u}{\partial t} + \nabla \cdot S = -J \cdot E \quad u = \frac{1}{8\pi} (E \cdot D + B \cdot H) \quad S = \frac{c}{4\pi} E \times H \]