These problems are due Friday, November 16.

1. Find the Dirichlet Green’s function for the inside of an infinitely long cylinder without \( z \) dependence using the eigenfunction method.

(a) Show that the eigenfunctions can be chosen to be

\[
\Psi_{mn}(r, \theta) = C_{mn} J_m \left( \frac{x_{mn}}{R} r \right) e^{im\theta}
\]

(b) Find the resulting Green’s function.

The next three problems ask you to find the Dirichlet Green’s function for the inside of the “soup can” defined by the surfaces \( z = 0, \ z = L, \ r = a \). You should be able to verify the following forms

2.

\[
G_D(r, r') = 4 \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} e^{im(\theta - \theta')} J_m \left( \frac{x_{mn}}{a} r \right) J_m \left( \frac{x_{mn}}{a} r' \right)
\]

\[
\times \sinh \left[ \frac{x_{mn}}{a} z \right] \sinh \left[ \frac{x_{mn}}{a} (L - z) \right]
\]

3.

\[
G_D(r, r') = 4 \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} e^{im(\theta - \theta')} \sin \left( \frac{n\pi z}{L} \right) \sin \left( \frac{n\pi z'}{L} \right)
\]

\[
\times \left[ I_m \left( \frac{n\pi a}{L} \right) K_m \left( \frac{n\pi r}{L} \right) - K_m \left( \frac{n\pi a}{L} \right) I_m \left( \frac{n\pi r}{L} \right) \right]
\]
4.

\[ G_D(r, r') = \frac{8}{L^2 \alpha^2} \sum_{m=-\infty}^{\infty} \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} e^{im(\theta - \theta')} \sin \left( \frac{k \pi z}{L} \right) \sin \left( \frac{k \pi z'}{L} \right) \frac{J_m \left( \frac{x_{mn} r}{a} \right) J_m \left( \frac{x_{mn} r'}{a} \right)}{\left( \frac{x_{mn} a}{a} \right)^2 + \left( \frac{k \pi L}{L} \right)^2} f_{m+1}^2 (x_{mn}) \]

As you can guess from the form of the solutions, 2. and 3. are done with “patchwork,” and 4. is done with an eigenfunction expansion.