Ph 621 Homework

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This assignment is due Monday, March 5.

1. A particle slides on a pair of frictionless planes, which therefore make a kind of oscillator as shown in Figure 1. In order to make this easy (or at least possible) to grade I would like you to adhere to the following notation. Use the variable $x$ for the horizontal axis and $q$ to be the distance measured along the plane, so that $q \cos \alpha = x$. Also use the constant $k \equiv mg \sin \alpha$, so the Hamiltonian is

$$H_0 = \frac{p^2}{2m} + kq = E$$

(a) Solve the action-angle problem for this situation and display the frequencies.

(b) Initially the displacement is $x_0$. What are the energy and frequencies of the oscillations?

Figure 1: The double sliding board
2. Now add a perturbation

\[ H_1 = \epsilon k |q| \]

and solve the problem with the canonical first-order perturbation theory that we went over in class.

3. You can get the exact solution to the problem by simply taking the zeroth-order solution and replacing everywhere \( k \to (1 + \epsilon)k \). Now test how good the perturbation theory answer is. Expand the exact solution in powers of \( \epsilon \) and show that the first-order correction is identical to the perturbation theory result.