

THE BALMER SPECTRUM OF ATOMIC HYDROGEN

OBJECTIVE: To observe the visible wavelengths of the light emitted by atomic hydrogen and to deduce the corresponding structure of the excited states of hydrogen.

READING: Krane, sections 6.4-6.5.

THEORY: When the atoms of a gas or vapor are excited by an electrical discharge or a flame, the atoms emit radiation at discrete wavelengths. These wavelengths are characteristic of the atoms and can be used to identify the composition of materials. The individual wavelengths or frequencies correspond to electrons within the atom changing from a state with initial energy E_i to a state with final energy E_f , so that the energy of the emitted photon is

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \quad (1)$$

The discrete emitted wavelengths thus imply that the energy levels of the electrons in the atom are restricted to certain discrete values.

A mathematical regularity of the wavelengths of a set of spectral lines emitted by hydrogen atoms was noticed by Balmer in 1885, and this set of spectral lines is known as the *Balmer series*. Balmer observed that the wavelengths of the hydrogen lines in the visible region of the spectrum could be expressed as

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad (2)$$

where R is the Rydberg constant ($1.0973732 \times 10^7 \text{ m}^{-1}$) and $n = 3, 4, 5, \dots$

The Bohr theory of the structure of the hydrogen atom, which was developed in 1913, gives the same results for the discrete allowed energy levels as the full quantum-mechanical calculation, which was developed more than a decade later:

$$E_n = -\frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^2} \quad (3)$$

Combining equations 1, 2, and 3 yields an expression for the Rydberg constant:

$$R = \frac{me^4}{64\pi^3 \epsilon_0^2 \hbar^3 c} \quad (4)$$

APPARATUS: To observe the spectral lines and determine their wavelengths, we use a diffraction grating, which passes the light through a large number of equally spaced parallel slits; the resulting pattern of interference maxima occurs when the light arrives in phase at the image plane. Bright light of the characteristic wavelengths (colors) can then be observed at those locations. The condition for constructive interference in first order is

$$\lambda = d \sin \theta \quad (5)$$

where d is the spacing between the slits and θ is the angle by which the light is deviated from its original path. The geometry of the experiment for observing the spectral lines with the grating is shown in Figure 1.

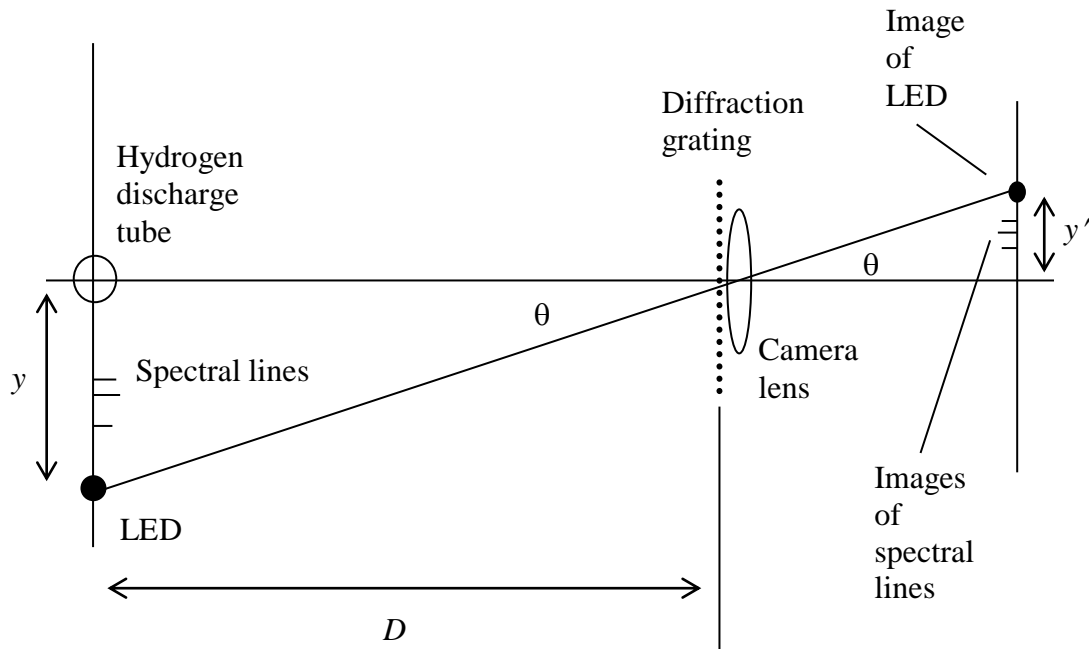


Figure 1

If the distance y of the spectral lines from the central axis of the apparatus were known, it would be possible to find θ from $\tan \theta = y/D$, where D is the distance between the diffraction grating and the object plane containing the H tube and the spectral lines. The spectral lines will be measured by photographing them with a digital camera and analyzing the resulting photo on a computer. Placing an LED in the object plane allows you to calibrate distances along the y direction. Knowing the distance y for the LED and determining the distance y' by counting pixels on the photographic image gives the necessary calibration (the distance/pixel ratio). By determining the distances y' in pixels for the images of the spectral lines, you can deduce their locations y and then their angular locations θ .

PROCEDURE:

WARNING: The power supply and discharge lamp operate at high voltage and have bare (uninsulated) contacts. Be careful not to touch these contacts.

The hydrogen discharge lamp should not be left on for long periods. Turn it on only when needed for viewing or photographing the spectrum.

Place the diffraction grating in front of the camera, oriented so that the spectral lines are spread horizontally. (Be sure the grating is oriented so that the rulings are vertical; usually that means that the label on the grating faces up.) Align and focus the camera so that you can see all of the spectral lines on one side of the tube and as many as possible on the other side. Place the LED at a convenient location to give you a reliable calibration (making y for the LED as large as possible) but not to interfere with any of the spectral lines.

Transfer the digital image to your computer and determine the locations in pixels of the spectral lines and the LED. From the known position of the LED, calculate the y coordinates of the spectral lines and determine the angle θ for each line. Average the results for identical lines on opposite sides of the axis and determine the best value for the wavelength of each line.

Estimate the uncertainty in each measurement of the wavelength, including the error in determining the image position from the computer and other geometrical errors that may be part of the measurement.

Knowing that the smallest value of n gives the largest wavelength, assign values of n to your measured wavelengths and plot $1/\lambda$ vs. $1/n^2$. Determine the slope and intercept, and interpret. Use your estimated uncertainty in the wavelengths to place uncertainty limits on the slope and intercept.

Determine the best value of the Rydberg constant R (and its uncertainty) from your data.

QUESTIONS:

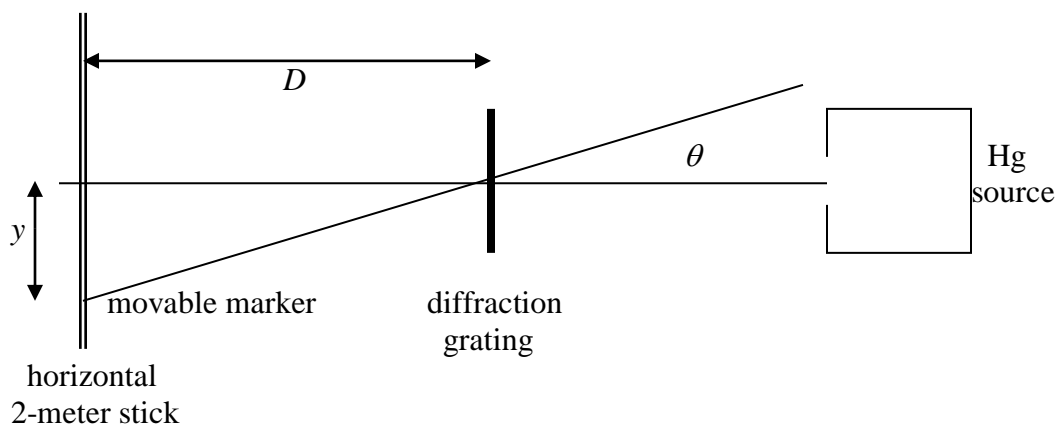
1. From the density of pixels on the digital image, can you make any conclusions about the relative intensities of the spectral lines? Identify the most intense lines and the least intense lines.
2. Suppose we repeated this experiment with a helium discharge tube instead of hydrogen. In the Bohr model, the electrostatic force on the electron in hydrogen is $e^2/4\pi\epsilon_0 r^2$, while in helium ($Z = 2$) the force is $2e^2/4\pi\epsilon_0 r^2$. Equation 2 applies only to hydrogen, but the transformation from hydrogen to helium can be accomplished, according to the force equations, by replacing e^2 with $2e^2$. How many spectral lines in the Balmer series of helium would be in the visible region of the spectrum? Assume that the Balmer series in helium likewise involves the factor $(1/2^2 - 1/n^2)$ with $n = 3, 4, 5, \dots$

ATOMIC EMISSION SPECTRUM

OBJECTIVE: To observe the visible wavelengths of the light emitted by a non-hydrogen atomic element and deduce the relative structure of the observed states.

PROCEDURE: In this activity you will use a diffraction grating to measure the wavelengths of the light produced by a mercury vapor lamp. In this experiment you will use a grating with a very small slit separation (600 slits per mm) and a large number of slits (more than 10,000).

Note that the angles in this case will be large enough that the small-angle approximation will not be valid. You must calculate the angle from the inverse tangent of y/D , and use $\sin \theta$ in your calculations.



Sight through the mark on the grating at one of the mercury spectral lines and align a movable marker with your line of sight. You will get better results if you keep your eye focused on the spectral line and allow the image of the movable marker to remain unfocused. Do this for each color visible in the spectrum. Record the position of each spectral line marker on the data sheet. Locate the zero position on the meter stick from the direct image of the Hg source. Use the two readings from either side of the center to get an average value of θ for each spectral line. Calculate the wavelength of each of the spectral lines and compare with the accepted values.

Grating spacing $d =$ _____ Position of central image: _____

Grating to meter stick distance $D =$ _____

Line	Accepted λ	y (left side)	y (right side)	θ avg	$\lambda = d \sin \theta$	% diff.
Red	689 nm					
Yellow	578 nm					
Green	546 nm					
Blue	492 nm					
Indigo	436 nm					

QUESTIONS:

1. What is the advantage of using a large number of slits as opposed to just two slits, assuming the separation between adjacent slits is the same?
2. How would your measurements above change if the Hg source were moved further back from the grating? Explain briefly.
3. How can this type of spectral analysis be used to determine the composition (percent amount of each element) of distant stars?