Problem 1.1 Power series (quiz)  Write down the first two non-zero terms in the power expansion of the following functions.

a) \[ \frac{1}{1 - x} \]

b) \[ \log(1 + x) \]

c) \[ \cos(x) \]

d) \[ \sin(x) \]

Problem 1.2 Limiting cases (quiz)  For each of the following expressions, find the limiting case when \( x \ll 1 \).

a) \[ \tan x \]

b) \[ \frac{\sin(x^2)}{\sin x} \]

c) \[ \frac{\sin x}{x} \]

d) \[ \ln(1 + x^2) \]
Problem 1.3 Finding entropy (quiz)  Given the following expressions for \(dQ\) and \(T\) for a quasistatic process, solve for the change in entropy from \(t = 0\) to \(t = t_f\), where \(t\) is time. You may take any other variables used to be constant (i.e. independent of time).

a) \[dQ = Pdt, \quad T = T_0 + Kt\]

b) \[dQ = Pdt, \quad T = T_0\]

c) \[dQ = -P e^{\frac{t}{t_0}} dt, \quad T = Ke^{-\frac{t}{2t_0}}\]

Problem 1.4 Heat and work (quiz)  For each of the following processes, solve for the heat or work done.

a) A system expands from volume \(V_0\) to volume \(V_f\). During this process the pressure is given by

\[p = \frac{Nk_B T}{V}\]

(5.15)

where \(k_B T\) and \(N\) are constant. How much work does the system do on its environment?

b) A system is heated from initial entropy \(S_0\) to final entropy \(S_f\). During this process the temperature given by

\[T = T_0 + \frac{S - S_0}{C_V}\]

(5.16)

where \(T_0\) and \(C_V\) are constants. How much energy is transferred into the system by heating during this process?

c) A system expands from volume \(V_0\) to volume \(V_f\). During this process the pressure is given by

\[p = p_0 \left(\frac{V_0}{V}\right)^\gamma\]

(5.17)

where \(p_0\) is the initial pressure. How much work does the system do on its environment?
Problem 1.5 Checking for reasonableness  For each of the following equations, check whether it could possibly make sense. You will need to check both dimensions and whether the quantities involved are intensive or extensive. For each equation, explain your reasoning.

You may assume that quantities with subscripts such as $V_0$ have the same dimensions and intensiveness/extensiveness as they would have without the subscripts.

a)  \[ p = \frac{N^2 k_B T}{V} \]

b)  \[ p = \frac{N k_B T}{V} \]

c)  \[ U = \frac{3}{2} k_B T \]

d)  \[ U = -N k_B T \ln \frac{V}{V_0} \]

e)  \[ S = -k_B \ln \frac{V}{V_0} \]

f)  \[ S = -k_B \ln \frac{V}{N} \]

Problem 1.6 Extensive internal energy  Consider a system which has an internal energy defined by:

\[ U = \gamma V^\alpha S^\beta \]  \hspace{1cm} (5.18)

where $\alpha$, $\beta$ and $\gamma$ are constants. The internal energy is an extensive quantity. What constraint does this place on the values $\alpha$ and $\beta$ may have?

Problem 1.7 Adiabatic compression  Consider the adiabatic expansion of a simple ideal gas. The internal energy is given by

\[ U = C_v T \]  \hspace{1cm} (5.19)

where you may take $C_v$ to be a constant—although for a polyatomic gas such as oxygen or nitrogen, it is temperature-dependent. The ideal gas law

\[ pV = N k_B T \]  \hspace{1cm} (5.20)

determines the relationship between $p$, $V$ and $T$. You may take the number of molecules $N$ to be constant.

a)  Use the first law to relate the inexact differential for work to the exact differential $dT$ for an adiabatic process.

b)  Find the total differential $dT$ where $T$ is a function $T(p, V)$.

c)  In the previous two sections, we found two formulas involving $dT$. Use the additional definition of work $dW = -pdV$ to solve for the relationship between $p$, $dp$, $V$ and $dV$ for an adiabatic process.

d)  Integrate the above differential equation to find a relationship between the initial and final pressure and volume for an adiabatic process.
Problem 1.8 A bottle in a bottle  

The internal energy of helium gas at temperature $T$ is to a very good approximation given by

$$U = \frac{3}{2} N k_B T$$  \hspace{1cm} (5.21)

Consider a very irreversible process in which a small bottle of helium is placed inside a large bottle, which otherwise contains vacuum. The inner bottle contains a slow leak, so that the helium leaks into the outer bottle. The inner bottle contains one tenth the volume of the outer bottle, which is insulated. What is the change in temperature when this process is complete? How much of the helium will remain in the small bottle?