

Symmetries and Idealizations Homework 1

Due Wednesday 9/30

Problem 1.1 Dimensions (practice) When physicists calculate the value of a physical quantity from an equation, they pay particular attention to the units involved. A force of 2 is ill-defined, a force of 2 Newtons is clear. When physicists want to check the plausibility of an equation, without worrying exactly about which set of units will be used (e.g. Newtons vs. pounds vs. dynes), they often look at the “dimensions” of the physical quantities involved. “Dimension” refers to the powers of the basic physical quantities: length (L), time (T), mass (M), and charge (C), that make up the physical quantity. For example, since force is mass times acceleration, the dimensions of force are ML/T^2 . Find the dimensions of electrostatic potential energy. Also, find the dimensions of electrostatic potential.

Solution to problem 1.1 Dimensions For electrostatic potential energy (note: the brackets indicate that we’re talking about the dimensions of the quantity):

$$[U] = \frac{ML^2}{T^2}$$

For electrostatic potential:

$$[V] = \frac{ML^2}{QT^2}$$

Problem 1.2 Potential vs. Energy In this course, two of the primary examples we will be using are the force due to gravity and the force due to an electric charge. Both of these forces vary like $1/r^2$, so they will have many, many similarities. Most of the calculations we do for the one case will be true for the other. But there are some extremely important differences:

- a) Find the value of the electric potential energy of a system consisting of a hydrogen nucleus and an electron separated by the Bohr radius. Find the value of the gravitational potential energy of the same two particles at the same radius. Use the same system of units in both cases. Compare and contrast the two answers.

Solution:

Bohr radius: $a_0 \approx 0.0529$ nm

Proton charge: $Q \approx 1.60 \times 10^{-19}$ C

$\frac{1}{4\pi\epsilon_0} \approx 9.0 \times 10^9$ Nm²/C²

Gravitational constant: $G \approx 6.67 \times 10^{-11}$ Nm²/kg²

Proton mass: $M \approx 1.67 \times 10^{-27}$ kg

Electron charge: $q \approx -1.60 \times 10^{-19}$ C

Electron mass: $m \approx 9.11 \times 10^{-31}$ kg

Therefore:

$$U_{\text{elec}} = qV_{\text{elec}} \quad (1.1)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{qQ}{a_0} \quad (1.2)$$

$$\approx -4.35 \times 10^{-18} \text{ J} \quad (1.3)$$

$$U_{\text{grav}} = mV_{\text{grav}} \quad (1.4)$$

$$= -G \frac{mM}{a_0} \quad (1.5)$$

$$\approx -1.92 \times 10^{-57} \text{ J} \quad (1.6)$$

The potential energy due to gravity in an atom is 28 orders of magnitude smaller than the electromagnetic force!

- b) Find the value of the electric potential due to the nucleus of a hydrogen atom at the Bohr radius. Find the gravitational potential due to the nucleus at the same radius. Use the same system of units in both cases. Compare and contrast the two answers.

Solution:

See constants above. Therefore:

$$V_{\text{elec}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_0} \approx 27.2 \text{ J/C} \quad (1.7)$$

$$V_{\text{grav}} = -G \frac{M}{r_0} \approx -2.11 \times 10^{-27} \text{ J/kg} \quad (1.8)$$

Even though these are measured in the same system of units, they are not in the same units and cannot be compared. (Notice that qV_{elec} is **not** the binding energy of the electron. The electron in the Bohr model also has kinetic energy.)

- c) Think of and briefly discuss at least one other fundamental difference between electromagnetic and gravitational systems. Hint: Why are we bound to the earth gravitationally, but not electromagnetically?

Solution:

One difference is that charges of both signs exist, so that electromagnetic forces can cancel each other. The earth is essentially neutral, so that we do not feel an electromagnetic attraction to the earth. Gravitational forces, however, can only add, so that the total force due to lots of mass can be very large.

Another difference is that masses always attract, so that the gravitational potential is negative. Whereas, positive charges repel each other so that the electric potential of a positive charge is positive.

Problem 1.3 Trig parameters Make sketches of the following functions, by hand, all on the same axes. Briefly describe, using good scientific writing that includes both words and equations, the role that the number two plays in the shape of each graph:

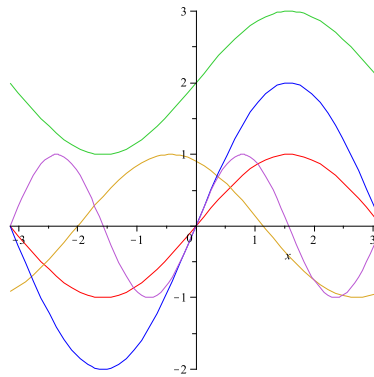
$$y = \sin x \quad (1.9)$$

$$y = 2 + \sin x \quad (1.10)$$

$$y = \sin(2 + x) \quad (1.11)$$

$$y = 2 \sin x \quad (1.12)$$

$$y = \sin 2x \quad (1.13)$$



Solution to problem 1.3 Trig parameters

| Color | Function | Description |
|--------|---------------|--|
| Red | $\sin x$ | the original function |
| Green | $2 + \sin x$ | shifts the function 2 units up along the y-axis. |
| Gold | $\sin(2 + x)$ | shifts the function right 2 units along the x-axis. |
| Blue | $2 \sin x$ | increases the amplitude of the function by 2 units. |
| Purple | $\sin 2x$ | squishes the function so that the period of the function is half as large as before. |

Problem 1.4 The θ function The function $\theta(x)$ (the *Heaviside* or *unit step function*) is defined as:

$$\theta(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

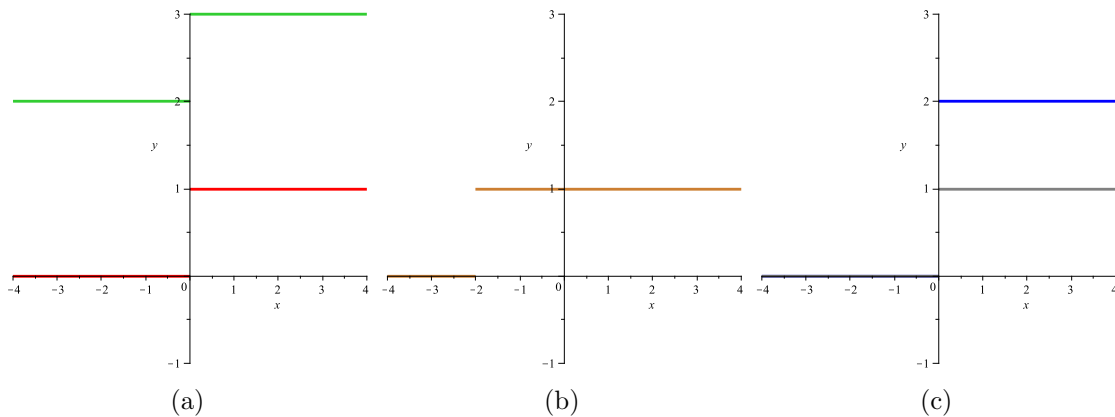


Figure 1.1: Solution to Problem 1.4. (a) in red, $\theta(x)$, the original function, and in green, $2 + \theta(x)$, shifts the function 2 units up along the vertical axis. (b) in gold, $\theta(2 + x)$, shifts the function right 2 units along the horizontal axis. (c) in blue, $2\theta(x)$, increases the amplitude of the function by two units, and in purple, $\theta(2x)$, nothing happens.

(see p. 352 of Riley, Hobson & Bence). This function is discontinuous at $x = 0$ and is generally taken to have a value of $\theta(0) = 1/2$.

Make sketches of the following functions, by hand, on axes with the same scale and domain. Briefly describe, using good scientific writing that includes both words and equations, the role that the number two plays in the shape of each graph:

$$y = \theta(x) \tag{1.14}$$

$$y = 2 + \theta(x) \tag{1.15}$$

$$y = \theta(2 + x) \tag{1.16}$$

$$y = 2\theta(x) \tag{1.17}$$

$$y = \theta(2x) \tag{1.18}$$

Solution:

See Figure 0.1.

Problem 1.5 Triangle function (challenge) Consider the function:

$$f(x) = 3x\theta(x)\theta(1-x) + (6-3x)\theta(x-1)\theta(2-x)$$

Make sketches of the following functions, by hand, on the axes with the same scale and domain. Briefly describe, using good scientific writing that includes both words and equations, the role that the number two plays in the shape of each graph:

$$y = f(x) \tag{1.19}$$

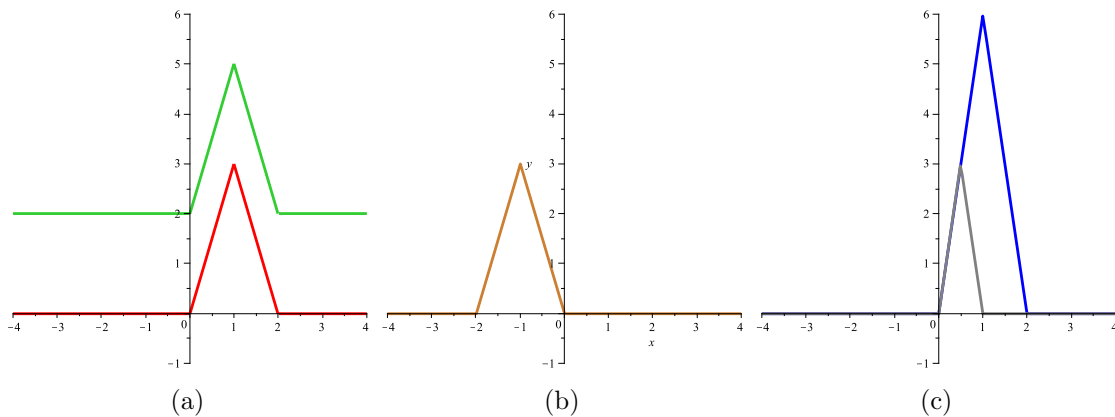


Figure 1.2: Solution to Problem 1.5. (a) in red, $f(x)$, the original function, and in green, $2 + f(x)$, shifts the function 2 units up along the vertical axis. (b) in gold, $f(2+x)$, shifts the function right 2 units along the horizontal axis. (c) in blue, $2f(x)$, increases the amplitude of the function by two units, and in purple, $f(2x)$, nothing happens.

$$y = 2 + f(x) \tag{1.20}$$

$$y = f(2 + x) \tag{1.21}$$

$$y = 2f(x) \tag{1.22}$$

$$y = f(2x) \tag{1.23}$$

Solution:

See Figure 0.2.