

Symmetries and Idealizations Homework 5

Due Wednesday 10/14

Problem 5.1 Directional derivative (practice) Imagine you're standing on a landscape with a local topology described by the function $f = kx^2y$, where $k = 20 \frac{m}{km^3}$ is a constant, x and y are east and north coordinates, respectively, with units of kilometers. You're standing at the spot (3 km, 2 km) and there is a cottage located at (1 km, 2 km). At the spot you're standing, what is the slope of the ground in the direction of the cottage? Plot the function f in Maple. Does your result makes sense with the picture?

Problem 5.2 Infinite disk Find the electrostatic potential due to an infinite disk, using your results from the finite disk problem.

Problem 5.3 A point charge

- Give an expression for the electric potential $V(\vec{r})$ at a point \vec{r} due to a point charge located at \vec{r}' .
- Give an expression for the electric field $\vec{E}(\vec{r})$ at a point \vec{r} due to a point charge located at \vec{r}' .
- Working in your favorite coordinate system, compute the gradient of V .
- Write several sentences comparing your answers to the last two questions.

Problem 5.4 Line sources

- Find the electric field around an infinite, uniformly charged, straight wire, starting from the expression for the electrostatic potential that we found in class:

$$V(\vec{r}) = \frac{2\lambda}{4\pi\epsilon_0} \ln \frac{r_0}{r}$$

- Find the electric field around a finite, uniformly charged, straight wire, at a point a distance r straight out from the midpoint, starting from the expression for the electrostatic potential that we found in class:

$$V(\vec{r}) = \frac{\lambda}{4\pi\epsilon_0} \left[\ln \left(L + \sqrt{L^2 + r^2} \right) - \ln \left(-L + \sqrt{L^2 + r^2} \right) \right]$$

- Find the electric field around an infinite, uniformly charged, straight wire, starting from Coulomb's Law.
- Find the electric field around a finite, uniformly charged, straight wire, at a point a distance r straight out from the midpoint, starting from Coulomb's Law.