Symmetries and Idealizations Homework 3
Due Wednesday 10/7

Problem 3.1 Ice cream mass (practice) Use integration to find the total mass of ice cream in a packed cone (both cone and hemisphere of ice cream on top).

Problem 3.2 Total charge (practice) For each case below, find the total charge.

a) A positively charged (dielectric) spherical shell of inner radius a and outer radius b with a spherically symmetric internal charge density $\rho(\vec{\boldsymbol{r}}) = \alpha\, 3e^{(kr)^3}$

b) A positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density $\rho(\vec{\boldsymbol{r}}) = \alpha\, \frac{1}{r}\, e^{kr}$.

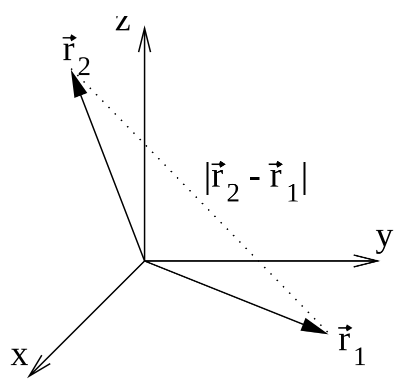
Problem 3.3 Quadrupole

a) A linear quadrupole is a series of three charges in a line, in this case, along the z-axis. Charges $+Q$ at $z = \pm D$ and charge $-2Q$ at $z = 0$. Find the electrostatic potential at a point P in the x,y-plane at a distance r from the center of the quadrupole.

b) Assume $r \gg D$. Find the first two non-zero terms of a Laurent series expansion to the electrostatic potential you found in the first part of this problem.

Problem 3.4 Curvilinear distance

a) Find the distance $|\vec{r}_2 - \vec{r}_1|$ between the point $\vec{r}_1 = (x_1, y_1, z_1)$ and the point $\vec{r}_2 = (x_2, y_2, z_2)$ in rectangular coordinates.



b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{r_2^2 + r_1^2 - 2r_1 r_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2}$$

c) Show that this same distance written in spherical coordinates is:

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{r_2^2 + r_1^2 - 2r_1 r_2 \left[\sin\theta_2 \sin\theta_1 \cos(\phi_2 - \phi_1) + \cos\theta_2 \cos\theta_1\right]}$$

d) Now assume that $\vec{r}_1$ is in the x-y plane. Simplify the previous two formulas.

e) **CHALLENGE:**[1] Find the distance $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$ between the point $\vec{\boldsymbol{r}}$ and the point $\vec{\boldsymbol{r}}'$ in terms of the magnitudes of $\vec{\boldsymbol{r}}$ and $\vec{\boldsymbol{r}}'$ and γ, the angle between them. (Do **not** choose a coordinate system.) Then assuming that $\vec{\boldsymbol{r}} \gg \vec{\boldsymbol{r}}'$, find a series expansion for $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$, correct to fourth order. This expansion is the basis of multipole expansions, used in both electromagnetic theory and quantum mechanics.

Problem 3.5 Mass of a slab Determine the total mass of each of the slabs below.

a) A square slab of side length L with thickness h, resting on a table top at $z = 0$, whose mass density is given by $\rho = A\pi \sin(\pi z/h)$.

b) A square slab of side length L with thickness h, resting on a table top at $z = 0$, whose mass density is given by

$$\rho = 2A\Big(\Theta(z) - \Theta(z - h)\Big) \tag{8.4}$$

c) An infinitesimally thin square sheet of side length L, resting on a table top at $z = 0$, whose surface density is given by $\sigma = 2Ah$.

d) An infinitesimally thin square sheet of side length L, resting on a table top at $z = 0$, whose mass density is given by $\rho = 2Ah\,\delta(z)$.

e) Write several sentences comparing your answers. What units does A have?

[1] not required, but give it a try if you are not overwhelmed with the course.