Symmetries and Idealizations Homework 1

Due Wednesday 9/30

Problem 1.1 Dimensions (practice) When physicists calculate the value of a physical quantity from an equation, they pay particular attention to the units involved. A force of 2 is ill-defined, a force of 2 Newtons is clear. When physicists want to check the plausibility of an equation, without worrying exactly about which set of units will be used (e.g. Newtons vs. pounds vs. dynes), they often look at the "dimensions" of the physical quantities involved. "Dimension" refers to the powers of the basic physical quantities: length (L), time (T), mass (M), and charge (C), that make up the physical quantity. For example, since force is mass times acceleration, the dimensions of force are ML/T^2. Find the dimensions of electrostatic potential energy. Also, find the dimensions of electrostatic potential.

Problem 1.2 Potential vs. Energy In this course, two of the primary examples we will be using are the force due to gravity and the force due to an electric charge. Both of these forces vary like $1/r^2$, so they will have many, many similarities. Most of the calculations we do for the one case will be true for the other. But there are some extremely important differences:

a) Find the value of the electric potential energy of a system consisting of a hydrogen nucleus and an electron separated by the Bohr radius. Find the value of the gravitational potential energy of the same two particles at the same radius. Use the same system of units in both cases. Compare and the contrast the two answers.

b) Find the value of the electric potential due to the nucleus of a hydrogen atom at the Bohr radius. Find the gravitational potential due to the nucleus at the same radius. Use the same system of units in both cases. Compare and contrast the two answers.

c) Think of and briefly discuss at least one other fundamental difference between electromagnetic and gravitational systems. Hint: Why are we bound to the earth gravitationally, but not electromagnetically?

Problem 1.3 Trig parameters Make sketches of the following functions, by hand, all on the same axes. Briefly describe, using good scientific writing that includes both words and equations, the role that the number two plays in the shape of each graph:

$$y = \sin x \tag{1.1}$$
$$y = 2 + \sin x \tag{1.2}$$
$$y = \sin(2 + x) \tag{1.3}$$
$$y = 2 \sin x \tag{1.4}$$
$$y = \sin 2x \tag{1.5}$$

Problem 1.4 The θ function The function $\theta(x)$ (the *Heaviside* or *unit step function*) is a defined as:

$$\theta(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

(see p. 352 of Riley, Hobson & Bence). This function is discontinuous at $x = 0$ and is generally taken to have a value of $\theta(0) = 1/2$.

Make sketches of the following functions, by hand, on axes with the same scale and domain. Briefly describe, using good scientific writing that includes both words and equations, the role that the number two plays in the shape of each graph:

$$y = \theta(x) \tag{1.6}$$
$$y = 2 + \theta(x) \tag{1.7}$$
$$y = \theta(2 + x) \tag{1.8}$$
$$y = 2\theta(x) \tag{1.9}$$
$$y = \theta(2x) \tag{1.10}$$

Problem 1.5 Triangle function (challenge) Consider the function:

$$f(x) = 3x\,\theta(x)\,\theta(1 - x) + (6 - 3x)\,\theta(x - 1)\,\theta(2 - x)$$

Make sketches of the following functions, by hand, on the axes with the same scale and domain. Briefly describe, using good scientific writing that includes both words and equations, the role that the number two plays in the shape of each graph:

$$y = f(x) \tag{1.11}$$
$$y = 2 + f(x) \tag{1.12}$$
$$y = f(2 + x) \tag{1.13}$$
$$y = 2f(x) \tag{1.14}$$
$$y = f(2x) \tag{1.15}$$