Central Forces Homework 4

Due 5/17/19, 4 pm

PRACTICE:

NONE.

REQUIRED:

1. (6 points) A mass μ, under the influence of a central-force field, moves in a logarithmic spiral orbit given by $r = ke^{\alpha\theta}$, where k and θ are constants. Determine the force law of this central-force field.

2. (2, 6 points) A particle moves in a force field described by the Yukawa potential

$$V(r) = -\frac{k}{r}e^{-r/a},$$

where k and a are positive. $V_1(r)$ is the first-approximation of the Yukawa potential in r/a.

   (a) Determine $V_1(r)$.

   (b) Determine the trajectory $r(\phi)$ of a mass μ in a bound orbit of $V_1(r)$.

3. (6, 6 points) The diagram (Figure 1) below shows a two-stage transfer scheme that brings a spacecraft of the sun (the sphere in the middle of the diagram) from a lower circular orbit into a higher one. The transfer orbit (solid yellow) is one half of an elliptic orbit that touches both the lower circular orbit the spacecraft wishes to leave (green and labeled 1 on diagram) and the higher circular orbit that it wishes to reach (red and labeled 3 on diagram). In stage 1, the spacecraft's engine is fired to instantaneously change its velocity so that it will follow the elliptical orbit. In stage 2, when the spacecraft has reached its destination orbit, the spacecraft's engine is again fired to instantaneously change its velocity to change the elliptic orbit to the larger circular one. In the following calculation, you may assume: (1) the sun is stationary and provides a gravitational potential $-k/r$; (2) the mass of the spacecraft is m and the mass of consumed fuel is negligible.

   (a) Determine Δv and $\Delta v'$, the velocity change at stage 1 and 2 respectively.

   (b) Calculate the transfer time between stage 1 and 2.

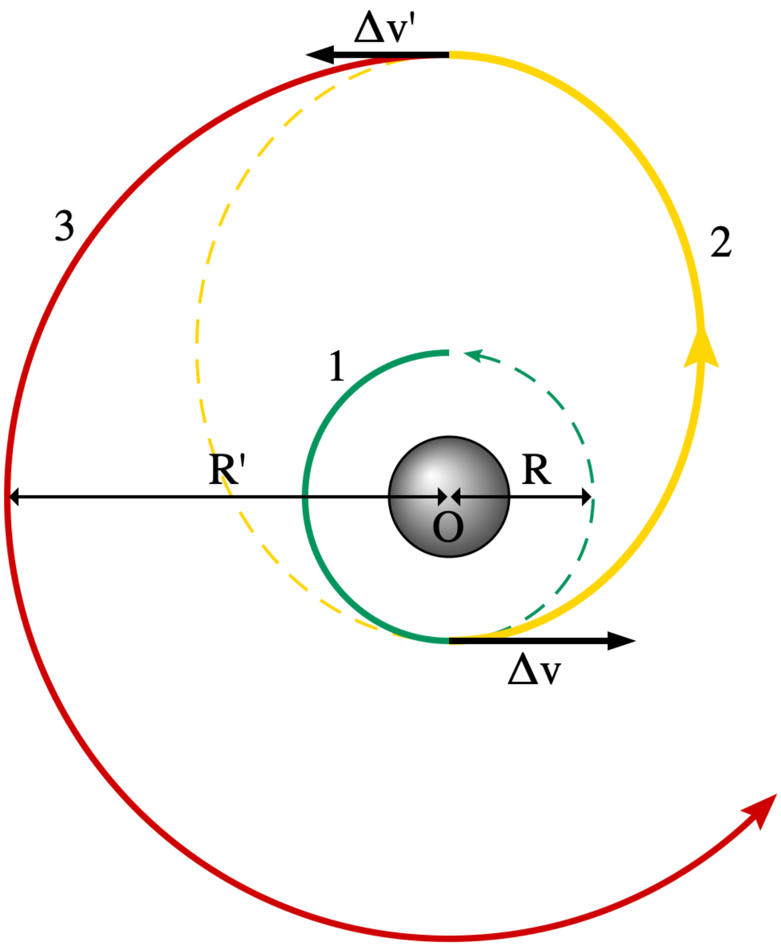

Figure 1: The transfer orbit, labelled 2, from an orbit (1) to a higher orbit (3).