Central Forces Homework 1

Due 5/8/19, 4 pm

PRACTICE:

1. In each of the following sums, shift the index $n \to n+2$. Don't forget to shift the limits of the sum as well. Then write out all of the terms in the sum (if the sum has a finite number of terms) or the first five terms in the sum (if the sum has an infinite number of terms) and convince yourself that the two different expressions for each sum are the same:

   (a)
   $$\sum_{n=0}^{3} n$$

   (b)
   $$\sum_{n=1}^{5} e^{in\phi}$$

   (c)
   $$\sum_{n=0}^{\infty} a_n n(n-1) z^{n-2}$$

REQUIRED:

Sensemaking: For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

1. (3, 3, 3 pts)

   Recurrence Relations

   For each of the problems below, suppose you have been solving a differential equation using power series methods around the indicated point. You find the indicated recurrence relation. Write out the first four nonzero terms in the power series expansion. If the recurrence relation allows two solutions, write out the first five nonzero terms in each such solution.

   (a) In an expansion around the point $z = 1$, the recurrence relation is:
   $$a_{n+1} = \frac{1}{n+1}\, a_n$$

   (b) In an expansion around the point $z = 0$, the recurrence relation is:
   $$a_{n+2} = -\frac{(5-n)(6+n)}{(n+2)(n+1)}\, a_n$$

(c) In an expansion around the point $z = 0$, the recurrence relation is:

$$a_{n+2} = -\frac{(3-n)}{(n+2)(n+1)}\, a_n$$

2. (7, 2 pts)

   Consider the differential equation $y'' - 2y' + y = 0$.

   (a) Use the power series method to find the first six terms in each of two independent solutions to this differential equation.

   (b) Solve this differential equation using a different method and show that your answers are the same as part a.

3. (7, 2, 2pts)

   Consider the differential equation $y'' = \frac{2}{(1-x)^2} y$.

   (a) Use a power series expanded about $x = 0$ to find the first six terms in each of two independent solutions to this differential equation.

   (b) For what values of x do each of your power series solutions converge?

   (c) Suppose you were to subtract one of your two solutions from the other solution. Is the resulting function still a solution to the original differential equation? Explain.