

PH 422: Day 12

34 Dot Products and Components

A field can be expressed in many different coordinate systems. For example, in rectangular coordinates, the electric field is

$$\vec{\mathbf{E}} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$$

Similarly, in cylindrical coordinates,

$$\vec{\mathbf{E}} = E_r \vec{\mathbf{r}} + E_\phi \hat{\boldsymbol{\phi}} + E_z \hat{\mathbf{z}}$$

What is the x -component of $\vec{\mathbf{E}}$? Surely just E_x . The r -component? E_r .

How could you find these components, given $\vec{\mathbf{E}}$? The (scalar) component of a vector field in a given direction is just the projection in that direction. Projections are dot products. Thus,

$$E_x = \vec{\mathbf{E}} \cdot \hat{\mathbf{i}}$$

and

$$E_r = \vec{\mathbf{E}} \cdot \hat{\mathbf{r}}$$

Given a surface, it makes sense to ask what the component E_\perp of the electric field is, perpendicular to the surface. By the same reasoning, we have

$$E_\perp = \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}$$

where $\hat{\mathbf{n}}$ is the unit normal to the surface. The component parallel to the surface, E_\parallel is more subtle, since there are an infinite number of directions parallel to the surface. One way around this problem is to speak of *vector components*, by attaching the direction to the scalar component. For instance

$$\vec{\mathbf{E}}_\perp = E_\perp \hat{\mathbf{n}}$$

We can now define the parallel (vector) component of $\vec{\mathbf{E}}$ as the vector that is left after the perpendicular component is subtracted

$$\vec{\mathbf{E}}_\parallel = \vec{\mathbf{E}} - \vec{\mathbf{E}}_\perp$$

from which the magnitude $E_\parallel = |\vec{\mathbf{E}}_\parallel|$ can be computed using the Pythagorean Theorem if desired.

35 Boundary conditions on electric fields

How does the electric field behave near a charged surface? There is no obvious reason for the electric field to be the same on both sides of the surface. Using an infinitesimally small Gaussian surface and an infinitesimally small Amperian loop, you should show that \vec{E}_{\parallel} is continuous, while the discontinuity in \vec{E}_{\perp} is proportional to the surface charge density (and is therefore zero for continuous media), i.e.

$$\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{n}$$

You will need to argue that the surface and/or loop are small enough that the field is effectively constant, except, of course, that there might be abrupt discontinuous changes where the charge resides.

36 Boundary conditions on magnetic fields

Given a current-carrying surface, it makes sense to ask what the component B_{\perp} of the magnetic field is perpendicular to the surface, which is $B_{\perp} = \vec{B} \cdot \hat{n}$, where \hat{n} is the unit normal to the surface. The component parallel to the surface, B_{\parallel} , is more subtle, since there are an infinite number of directions parallel to the surface. However, since for a current-carrying surface there is a preferred direction in the surface, namely the direction of the current \vec{K} , we can distinguish between the component in the surface and parallel to the current, called $B_{\parallel\parallel}$, and the component in the surface but perpendicular to the current called $B_{\parallel\perp}$.

Using an infinitesimally small Gaussian surface and an infinitesimally small Amperian loop, you should show that B_{\perp} and $B_{\parallel\parallel}$ are continuous, while the discontinuity in $B_{\parallel\perp}$ is proportional to the (local) surface current density. These conditions can be combined in the equation

$$\vec{B}_{\text{above}} - \vec{B}_{\text{below}} = \mu_0 (\vec{K} \times \hat{n})$$