PH 422: Day 10

31 Differential form of Ampère's Law

Recall that Ampère's Law says that

$$\oint_{\text{loop}} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}} = \mu_0 \, I_{\text{inside}}$$

But the enclosed current is just

$$I_{\text{inside}} = \int_{\text{inside}} \vec{\boldsymbol{J}} \cdot d\vec{\boldsymbol{A}}$$

so we have

$$\oint_{\text{loop}} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}} = \mu_0 \int_{\text{inside}} \vec{\boldsymbol{J}} \cdot d\vec{\boldsymbol{A}}$$

Putting this all together, Stokes' Theorem tells us that

$$\int_{\text{inside}} (\vec{\nabla} \times \vec{\boldsymbol{B}}) \cdot d\vec{\boldsymbol{A}} = \mu_0 \int_{\text{inside}} \vec{\boldsymbol{J}} \cdot d\vec{\boldsymbol{A}}$$

for *any* closed loop. This means that the integrands themselves must be equal, that is,

$$\vec{\nabla} \times \vec{\boldsymbol{B}} = \mu_0 \, \vec{\boldsymbol{J}}$$

This is the differential form of Ampère's Law, and is one of Maxwell's Equations. It states that the curl of the magnetic field at any point is just a measure of the current density there.

In the activity earlier this week, Ampère's Law was used to derive the magnetic field for a symmetric current distribution. The differential form of Ampère's Law makes it possible to go the other way.