Midterm

(Wednesday, November 1, 2017)

This is an open-book exam. The total number of points is 100. You have 50 minutes to complete all three problems.

1. **(30 pts)** The Hamiltonian of a two-state system is given by
   \[ H = E \left( |\varphi_1\rangle \langle \varphi_1| - |\varphi_2\rangle \langle \varphi_2| - i |\varphi_1\rangle \langle \varphi_2| + i |\varphi_2\rangle \langle \varphi_1| \right), \]
   where \( |\varphi_1\rangle, |\varphi_2\rangle \) form a complete and orthonormal basis; \( E \) is a real constant having the dimensions of energy.
   
   (a) Is \( H \) Hermitian?
   (b) If \( H \) is measured, what are the possible outcomes?
   (c) What are the possible (normalized) states of the system after this measurement?

2. **(40 pts)** Operators \( A \) and \( B \) are represented in some complete and orthonormal basis as follows:
   \[ A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} \quad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix} \]
   
   (a) Is \( \{A\} \) a C.S.C.O.? What about \( \{B\} \)? What about a set \( \{A,B\} \)?
   (b) Find a set of orthonormal kets that are simultaneous eigenkets of both \( A \) and \( B \).
      Specify the eigenvalues of \( A \) and \( B \) for each of the three eigenkets. Does your specification of eigenvalues completely characterize each eigenket?

3. **(30 pts)**
   
   (a) Are the following operators Hermitian, anti-Hermitian or neither? Show.
      i. \([X,P]\)
      ii. \([X^2,P]\)
   
   (b) Prove \( \text{Tr}(U^*AU) = \text{Tr}(A) \), where \( A \) is an arbitrary operator and \( U \) is a unitary operator.