Problems with classical theory and emergence of quantum mechanics

Black-body radiation

Material object that absorbs all incident radiation

Heated object radiates $\Rightarrow$ spectral energy density $U(v, T)$

Classical description:

Consider standing waves (eigenmodes) of a cavity

Solve electromagnetic wave equation with boundary conditions

Obtain $U(v, T) = \frac{8\pi}{c^3} v^2 k_B T$ (1900)

Rayleigh-Jeans law
Problem: blows up at $v \to \infty$!

(UV catastrophe)

Eventually: derivation assumed that the energy exchange between radiation and matter is continuous, i.e. any amount of energy can be exchanged. Which is wrong.

Another approach:

Wien (1889) ⇒ use thermodynamics and experimental Stefan–Boltzmann law

$E = \sigma T^4$, $\sigma = 5.67 \times 10^{-8} \text{W/m}^2\text{K}^4$

$U(v, T) = A v^3 e^{-\nu_v / T}$, where $A, \nu_v$ are adjustable parameters

Wien's formula (1894)

Problem: fits well only high-frequency data

Rayleigh–Jeans, $\propto v^2$

Planck, $\sim e^{-\nu_v / T}$
Planck (1900):

Laws of classical physics do not apply on an atomic scale.

- Radiating body consists of an enormous number of elementary oscillators vibrating at all possible frequencies.
- These oscillators are the source of the emitted radiation.
- The energy of an oscillator is quantized

\[ E = n \hbar \nu \]

\[ U (\nu, T) = \frac{8\pi \nu^2}{c^3} \frac{\hbar \nu}{e^{\hbar \nu / kT} - 1} \]

\[ \hbar = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \]

Exact explanation of the radiation process \(\Rightarrow\) Einstein light quanta
2. Photoelectric effect.

Experimental demonstration by Hertz (1887)

Experimental observations:

- Monochromatic light yields electrons of a definite energy.
- There is a threshold frequency $\nu_0$ at which electron emission starts, and it's instantaneous.
- At any frequency $\nu > \nu_0$, the energy of electrons is linearly proportional to the frequency of light.
- Kinetic energy of the electrons depends on the frequency, but not on the intensity of the beam.
- Increase in light intensity leads to the emission of more electrons, but does not change their energy.

Problem: In classical physics (wave theory) $\Rightarrow$

- The higher light intensity, the higher electron energy is expected.

Explanation: Einstein (1905) $\Rightarrow$

$$\frac{1}{2} m \nu^2 = h \nu - W \Rightarrow \text{binding energy of an electron in the metal}$$
Light radiation consists of a beam of corpuscles, the photons, of energy $h\nu$ and velocity $c$

$$c = 3.10^8 \frac{m}{s} \text{ in vacuum}$$

3. **Compton effect**

X-ray scattering by (free) free electrons

\[ \text{incident } \gamma, \beta \rightarrow E, \bar{E} \]

\[ \cos \theta = \frac{E}{\gamma} \]

\[ \Rightarrow \text{scattered } \gamma', \beta' \]

**Classical prediction**

- scattered light has the same frequency $\nu' = \nu$
- scattered $I' \sim I_{\text{inc}}$
- scattered intensity

**Experiment**

$\nu \neq \nu'$, $\Delta \lambda = \lambda' - \lambda = \frac{c}{\nu}\cdot \frac{\sin^2 \theta}{2}$

Compton scattering formula

$$\lambda_c = \frac{h}{m_e c} = 3.86 \times 10^{-13} \text{ m} \quad \text{Compton wavelength of the electron}$$

Photons collide with electrons like material particles
Other groundbreaking experiments

Franck-Hertz experiment (1913)

Triode filled with Hg vapor. Electrons are mostly from K to A through a grid G, to which a small countervoltage is applied.

At electron energies below 4.9 eV, the electrons don't "notice" the grid G and reach A ⇒ the current increases as the voltage increases. At 4.9 eV, Hg absorbs the electron's energy & the electron becomes slow and never reaches A (because of the grid G) ⇒ the current drops. The situation repeats at 4.9x2 = 9.8 eV, then at 4.9x3 = 14.7 eV, etc.

Demonstration of discreet energy levels in the Hg atom.
Stern-Gerlach experiment (1921)

Randomly oriented Ag atoms (neutral) in the ground state

Magnetic moment
\[ \vec{\mu} = \frac{e}{m_e c} \vec{S} \]  
\( \vec{S} \) = spin

\[ \Rightarrow \text{interacts with } \vec{B} \Rightarrow W = -\vec{\mu} \cdot \vec{B} \]

\( W = -\vec{\mu} \cdot \vec{B} \)

\( \text{Force along } z \Rightarrow F_z = -\frac{\partial W}{\partial z} \approx \mu_z \frac{\partial B_z}{\partial z} \)

\( = \frac{e}{m_e c} s_z \frac{\partial B_z}{\partial z} \)

Why two components?

Only two possible values of \( S_z \)

\[ S_z = \pm \frac{\hbar}{2} \]
Question: Can this experiment be done with hydrogen atoms? If yes, in what state?

Answer: Yes, and it was done with \( \text{H} \)-atoms later. One needs to use \( \text{H} \)-atoms in their ground state (1S), so that the angular momentum \( l = 0 \), otherwise \( j = \sqrt{l^2 + \frac{1}{4}} \) would lead to a mess!

Question: Why not do this experiment with electrons?

Answer: Electrons are charged particles, so in a magnetic field the force would not be just \( \frac{\partial}{\partial t} (\mathbf{p} \cdot \mathbf{B}) \), but would also include the Lorentz force \( \mathbf{F} = e \mathbf{v} \times \mathbf{B} \) leading to a mess.

Young experiment

| Incident wave | \( \frac{1}{2} \) \( \frac{1}{2} \) |

\[ I(x) \neq I_1(x) + I_2(x) \]

CCD sensor

Intensity distribution from a single slit
Recall classical physics:

Treat light as a conventional plain wave, with electric fields

\[ E_1(\vec{r}) = E_1^0 e^{i(\vec{k}_1 \cdot \vec{r} - \omega t)} \]
\[ E_2(\vec{r}) = E_2^0 e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)} \]

for the beams 1 and 2.

Then, total \( \vec{E} = \vec{E}_1 + \vec{E}_2 \Rightarrow \)

intensity \( I = |\vec{E}(\vec{r})|^2 = |E_1^0|^2 + |E_2^0|^2 + 2E_1^0 E_2^0 \cos (\vec{k}_1 - \vec{k}_2) \cdot \vec{r} \)

interference term

What if we reduce the amount of light, so that only one photon at a time passes through a double-slit apparatus? \( \Rightarrow \) after a very long time see the same interference pattern (although we know that the photons could not have interacted with each other, since we let them through one at a time!)

\[ \text{CCD sensor} \]

\[ \text{Very long time} \quad \Rightarrow \quad \text{Wave-like behavior} \]

\[ \text{Very short time (random mess)} \quad \text{Particle-like} \]
Conclusion: light behaves simultaneously as a wave and as a flux of particles.

How do we interpret the interference pattern produced by a single photon source? It is a probability amplitude that the photon arrives at a given spot at a given time. So, in QM there are no definite trajectories (in contrast to classical mechanics) only probability to find a system in a certain state.

Consider another example.

\[ \vec{E}(r,t) = E_0 \vec{e}_p e^{i(kz - \omega t)} \]

(Propagates along z-axis)

It's linearly polarized along \( \vec{e}_p \), which is under an angle \( \Theta \) with respect to x-axis's.

After the polarizer \( \vec{E}'(r,t) = E'_0 \vec{e}_x e^{i(kz - \omega t)} \)
Light intensity $I' = |\vec{E}'|^2 = E_1^2 = \frac{E_0^2 \cos^2 \theta}{I_{\text{before polarizer}}}$

$= I_{\text{before polarizer}} \cos^2 \theta$  \[\text{Malus's law}\]

What if $I_{\text{before polarizer}}$ is weak enough, so that the photons reach the detector one by one? $\Rightarrow$ The detector can't register "a fraction of a photon". The photon either passes through the polarizer or does not pass! We do not know what photon will pass and which one won't $\Rightarrow$ We only know the corresponding probabilities. Our detector can give only certain privileged results $\Rightarrow$ eigen results. Each of these eigen results corresponds to an eigenstate. $\Rightarrow$ In this case we have two eigenstates:

- one is characterized by $\vec{\mathbf{p}} = \hat{\mathbf{x}}$ (pass) and another one - by $\vec{\mathbf{p}} = \hat{\mathbf{y}}$ (does not pass). If before the measurement the particle is in one of the eigenstates, the result of the measurement is certain: the detector will produce the corresponding eigenresult.
If the state before measurement is arbitrary only probabilities of obtaining different eigenvalues can be predicted:
\[ \hat{\varepsilon}_p = \hat{\varepsilon}_x \cos \Theta + \hat{\varepsilon}_y \sin \Theta \]
(present initial state in terms of the eigenstates)
Probability of "passing": \( \cos^2 \Theta \), "not passing": \( \sin^2 \Theta \)
Total probability: \( \cos^2 \Theta + \sin^2 \Theta = 1 \)

Such decomposition in QM is called "principle of spectral decomposition"

Note: our detector distinguishes only between the states \( \hat{\varepsilon}_x \) and \( \hat{\varepsilon}_y \) (photon detected or undetected, respectively), and info about our initial state \( \hat{\varepsilon}_p \) is contained only in probabilities to get an outcome \( \hat{\varepsilon}_x \) or outcome \( \hat{\varepsilon}_y \) \( \Rightarrow \) so, the measurement event distorts the system, "forcing" it to show only its eigenresults.

This is most bizarre and beautiful property of QM systems!!