Homework #9
(due Wednesday, December 4, 2019)

1. (10 pts) Consider 1D harmonic oscillator.
   Evaluate the following commutators:
   (a) \([P_{H}(t_1), P_{H}(t_2)]\)
   (b) \([X_{H}(t_1), X_{H}(t_2)]\)
   Explain the physical meaning of your results.

2. (20 pts) Consider a particle which behaves as 1D harmonic oscillator. Now imagine that your particle is also charged (has an electric charge q) and apply uniform electric field \(E\) along x-axis.
   (a) Find the allowed energy levels and corresponding eigenfunctions. **Hint: you don’t need to solve anything to be able to do it! Add the appropriate term in the Hamiltonian and see how you can reduce the problem to that of a regular harmonic oscillator we discussed.**
   (b) At \(t < 0\) the particle is in the ground state. At \(t = 0\) the electric field is suddenly turned off. What is the probability to find the particle in the ground state and in the first excited state?

3. Use WKB to estimate the transmission coefficient of an electron (a particle of mass \(m\) and energy \(E\)) undergoing cold emission from metal (i.e. moving in a following potential \(V(x)\)):
   \(V(x) = 0 \) at \(x < 0\) and \(V(x) = V_0 - \lambda x\) at \(x > 0\) (where \(V_0\) is the work function of the metal and the constant \(\lambda = eE\), where \(e\) is the charge of the electron and \(E\) is the applied electric field). In particular:
   (a) Sketch \(V(x)\)
   (b) Set up the integral you need to solve in the WKB treatment of the problem.
      What are the integration limits?
   (c) Calculate the integral and determine the transmission coefficient
(d) Comment on the dependence of the transmission on the energy of the electron and other parameters.