1. (20 pts) Consider a particle of mass $m$ in the attractive 1D delta potential given by
   
   $$V(x) = -V_0 \delta(x), \text{ where } V_0 > 0.$$ 

   (a) In the case of negative energies, show that this particle has only one bound state; find the binding energy and the wave function.

   (b) What is the probability that the particle remains bound when $V_0$ suddenly changes to $V_0'$?

   (c) Study the scattering case (i.e. $E > 0$) and calculate the transmission and reflection coefficients as a function of the wave number $k$ (or energy $E$).

2. (10 pts) Consider a particle moving in $V(x) = V_0$ at $0 < x < a$ and $0$ elsewhere, where $V_0 > 0$. Derive the reflection and transmission coefficients for this potential barrier for the cases of $E < V_0$ and $E > V_0$. (Basically, finish the problem we discussed in class.)

3. (20 pts) Consider a particle of mass $m$ in the 1D potential well given by
   
   $$V(x) = -V_0 \text{ if } |x| < a \text{ and } V(x) = 0 \text{ if } |x| > a, \text{ where } V_0 \text{ is a positive number}.$$ 

   (c) Write down the Schroedinger equation for the wave functions in three regions
      
      $(x < -a, -a < x < a, x > a)$

   (d) Write down a general form of the physically admissible solution

   (c) Find the energy spectrum of the bound states (you will encounter some transcendental equations – solve them graphically)
(d) How does the number of the bound states depend on the parameters of the well (i.e. $V_0$ and $a$)?

4. (20 pts) A particle of mass $m$ is subject to an attractive double-delta potential $V(x) = -V_0 \delta(x-a) - V_0 \delta(x+a)$, where $V_0 > 0$. Consider only the case of negative energies.

(a) Obtain the wave functions of the bound states. **Hint**: do not forget to use symmetry arguments !!

(b) Derive the eigenvalue equations (you should get two transcendent equations, one for the odd wavefunctions and one for the even wavefunctions)

(c) Specify the number of bound states and the limit of their energies. Is the ground state an even state or an odd state?

(d) Estimate the ground state energy for the limits $a \to 0$ and $a \to \infty$.

5. (20 pts) Consider 1D harmonic oscillator. Find the matrix element of the position operator $X$ (i.e. $x_{nm}$) using:

(a) $x$-representation (and therefore, Hermite polynomials)

(b) number representation (and therefore, creation and annihilation operators).

6. (10 pts) Consider 1D harmonic oscillator. By setting up an eigenvalue equation in the momentum space and direct comparison with that in the position space, infer the momentum space eigenfunctions $\Phi(p)$ (you don’t have to solve anything here !).

7. Reading assignment: Sakurai 2.4-2.5; also take a look at modern research utilizing scanning tunneling microscopy - Nature Physics 12, 92 (2016).