Homework #6  
(due Wednesday, November 13, 2019)

1. (10 pts) Show that $[\mathbf{R} \cdot \mathbf{P}, H] = 2 \mathbf{i} \hbar T - \mathbf{i} \hbar \mathbf{R} \cdot \nabla V$, where $\mathbf{R}$ is the position operator in 3D space, $\mathbf{P}$ is the momentum operator, $H$ is the Hamiltonian ($H = \frac{\mathbf{P}^2}{2m} + V(\mathbf{R})$), and $T$ is the kinetic energy operator ($T = \frac{\mathbf{P}^2}{2m}$).

2. (20 pts) A particle of mass $m$, which moves inside an infinite 1D potential well of length $a$, is described by the following wave function at $t = 0$:

$$\psi(x,0) = \frac{A}{\sqrt{a}} \sin \left( \frac{\pi x}{a} \right) + \sqrt{\frac{3}{5a}} \sin \left( \frac{3\pi x}{a} \right) + \sqrt{\frac{1}{5a}} \sin \left( \frac{5\pi x}{a} \right),$$

where $A$ is a real constant.

(a) Find $A$ so that $\psi(x,0)$ is normalized

(b) If measurements of the energy are carried out at $t=0$, what are the values that will be found and what are the corresponding probabilities?

(c) What is the average energy?

(d) Find the wave function $\psi(x,t)$ at a later time $t$

(e) Determine the probability of finding the system at a time $t$ in the state

$$\psi(x,t) = \sqrt{\frac{2}{a}} \sin \left( \frac{5\pi x}{a} \right) e^{-iE_dt/\hbar}$$

(f) The same as (e) but for the state $\chi(x,t) = \sqrt{\frac{2}{a}} \sin \left( \frac{2\pi x}{a} \right) e^{-iE_dt/\hbar}$

3. (20 pts) Sakurai 2.9.

4. (10 pts) Consider a particle of mass $m$ in the 1D potential well given by

$$V(x) = \begin{cases} 0 & \text{if } |x| < a \\ +\infty & \text{if } |x| > a \end{cases}$$

Let’s say that the particle is in the ground state (i.e. in the state with the lowest energy). Suddenly, the well symmetrically expands to twice its size. What is the probability to find the particle in the ground state of this new well?

5. Reading assignment: Sakurai 2.1-2.2, 2.4.