that if $\Delta\nu \ll \nu_0\sigma_v/c$, $\overline{g}(\nu)$ may be approximated by the Gaussian lineshape function

$$\overline{g}(\nu) = \frac{1}{\sqrt{2\pi}\,\sigma_D} \exp\left[-\frac{(\nu-\nu_0)^2}{2\sigma_D^2}\right], \tag{13.3-41}$$

where

$$\sigma_D = \nu_0 \frac{\sigma_v}{c} = \frac{1}{\lambda}\sqrt{\frac{kT}{M}}. \tag{13.3-42}$$

The full-width half-maximum (FWHM) Doppler linewidth $\Delta\nu_D$ is then

$$\Delta\nu_D = \sqrt{8\ln 2}\,\sigma_D \approx 2.35\,\sigma_D. \tag{13.3-43}$$

Compute the Doppler linewidth for the $\lambda_o = 632.8$ nm transition in Ne and for the $\lambda_o = 10.6\ \mu$m transition in CO_2 at room temperature, assuming that $\Delta\nu \ll \nu_0\sigma_v/c$. These transitions are used in He–Ne and CO_2 gas lasers, respectively.

Show that the maximum value of the transition cross section for the Gaussian lineshape function in (13.3-41) is

$$\sigma_0 = \frac{\lambda^2}{8\pi}\sqrt{\frac{4\ln 2}{\pi}}\,\frac{1}{t_{sp}\,\Delta\nu_D} \approx 0.94\,\frac{\lambda^2}{8\pi}\,\frac{1}{t_{sp}\,\Delta\nu_D}. \tag{13.3-44}$$

Compare with (13.3-35) for the Lorentzian lineshape function.

Many atom–photon interactions exhibit broadening that is intermediate between purely homogeneous and purely inhomogeneous. Such mixed broadening can be modeled by an intermediate lineshape function such as the Voight profile.

E. Enhanced Spontaneous Emission

All of the results presented thus far in Sec. 13.3 are predicated on the assumption that $\Delta\nu \gg \delta\nu$, i.e., that the atomic linewidth $\Delta\nu$ is far greater than the width of an electromagnetic mode $\delta\nu$. This condition is usually, but not always, obeyed. In the opposite limit, when the atomic linewidth is far smaller than the width of an electromagnetic mode (Fig. 13.3-12), an enhancement of the spontaneous emission probability density can be achieved, particularly in high-Q microcavities, as we proceed to demonstrate. The enhancement of spontaneous emission is desirable for the operation of certain photon sources, as discussed in Sec. 17.4.

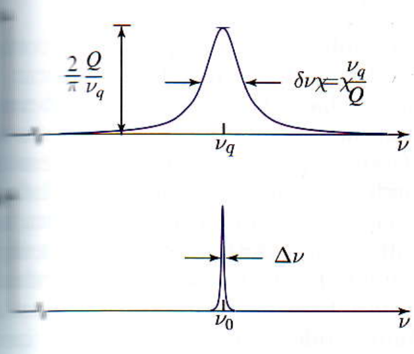

Figure 13.3-12 Spontaneous emission from an atom with normalized lineshape function $g(\nu)$ into a broader normalized Lorentzian cavity mode $\rho(\nu)$. The lineshape-function and cavity-mode center frequencies are designated by ν_0 and ν_q, respectively, while their widths are specified by $\Delta\nu$ and $\delta\nu$. We consider the case where $\nu_0 = \nu_q$ and $\Delta\nu \ll \delta\nu$.

Consider the spontaneous emission of an atom with resonance frequency ν_0 into an electromagnetic mode with center frequency $\nu_q = \nu_0$ in the regime $\Delta\nu \ll \delta\nu$, as portrayed in Fig. 13.3-12. In accordance with (13.3-11), when the dipole moment of the atom is aligned with the field direction of the mode, the probability density for spontaneous emission into a single cavity mode $\rho(\nu)$ is given by

$$P_{sp}^{max} = \int_0^\infty \frac{c}{V}\,\sigma_{max}(\nu)\,\rho(\nu)\,d\nu \approx \frac{c}{V}\,\frac{3\lambda^2}{8\pi t_{sp}}\,\rho(\nu_0)\int_0^\infty g(\nu)\,d\nu, \qquad (13.3\text{-}45)$$

since $\sigma_{max}(\nu) = 3\overline{\sigma}(\nu)$ and $\overline{\sigma}(\nu) = \lambda^2 g(\nu)/8\pi t_{sp}$, as provided in (13.3-10) and (13.3-15), respectively. Inasmuch as the lineshape function $g(\nu)$ is normalized, and the height of the normalized Lorentzian lineshape function of the cavity mode is $2Q/\pi\nu_q$, where $Q = \nu_q/\delta\nu$, we obtain

$$P_{sp}^{max} = \frac{1}{t_{sp}}\,\frac{3c\lambda^2}{8\pi V}\,\frac{2Q}{\pi\nu_q} = \frac{1}{t_{sp}}\cdot\frac{3}{4\pi^2}\,\frac{\lambda^3}{V}\,Q. \qquad (13.3\text{-}46)$$

The net result is an enhancement of the spontaneous emission probability density relative to that in free space by a quantity known as the **Purcell factor**:

$$\frac{P_{sp}^{max}}{P_{sp}} = \frac{3}{4\pi^2}\,\frac{\lambda^3}{V}\,Q. \qquad (13.3\text{-}47)$$

Purcell Factor

The Purcell factor in (13.3-47) exhibits the following features:

- The factor of 3 is a result of the alignment of the dipole moment of the atom and the field direction of the mode.
- The quantity λ^3/V, which is the ratio of the cubed wavelength to the cavity volume, is substantially enhanced in a microcavity.
- A high value of Q, i.e., a sharp cavity mode, enhances the Purcell factor; however, as Q increases, $\delta\nu = \nu_q/Q$ decreases, so that ultimately the condition $\Delta\nu \ll \delta\nu$ is violated.

As ν_0 deviates from ν_q, the height of the cavity mode at ν_0 becomes smaller and the enhancement of spontaneous emission ultimately becomes a suppression of spontaneous emission.

*F. Laser Cooling and Trapping of Atoms

It is often desirable to slow neutral atoms (**laser cooling**) and to trap them in a confined region of space (**atom trapping**). Ultracold atoms offer unparalleled accuracy for atomic clocks. Laser cooling and trapping can be achieved by arranging laser beams in such a way that they selectively impart photon momentum to a beam of atoms with well-regulated velocities (see Sec. 12.1D). Cooled and trapped atoms are essential components of **atom optics**, a field of research concerned with the manipulation of **matter waves**. Structured light waves often serve as atom-optical components; as in ordinary wave optics, reflection, refraction, diffraction, interference, and scattering of the matter waves are all observed. Matter-wave interferometry promises exceptionally sensitive measurements of local gravity anomalies. Cooled and trapped atoms are also important for the production of **Bose–Einstein condensates** (BECs), collections of atoms that are sufficiently slow and dense that their atomic wavefunctions overlap.

One of the simplest schemes for laser cooling relies on photons from a laser beam of narrow linewidth, with a center frequency tuned slightly below the atomic line center