SUMMARY OF THE "BIG A LITTLE A" ANALYSIS

When e^- concentration becomes high ($a < 3\text{-}4\,a_B^{eff}$) the Coulomb interaction holding e^-s onto the ionic cores becomes negligible.

We didn't consider e-e interactions explicitly, but these are equally strongly screened at high e^- concentrations.

↳ Justification for the non-interacting e^- approx for many situations.

We introduced an important new length scale

$$\boxed{a_B^{eff} = \frac{4\pi\epsilon\hbar^2}{m_{eff}\,e^2}}$$

PRACTICE

Calculate a_B^{eff} for conduction band e^-s in GaAs

Answer: $\approx 100\,\text{Å}$.

TOPOLOGICAL PHENOMENA IN ELECTRON TRANSPORT

1. Quantum Hall Effect (QHE)
2. ac-Josephsen Effect (a.c.JE)

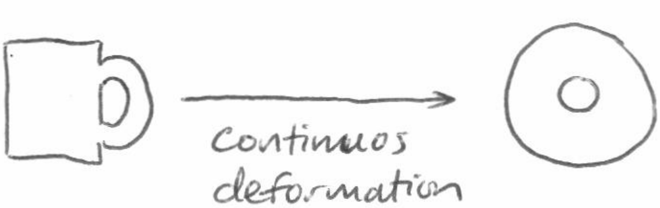
Not only beautiful e^- transport phenomena

Amongst the most beautiful physics discovered in last 40 years

1973 Nobel Prise for Josephson
1985 Nobel Prize for QHE (Klitzing)
1998 Nobel Prize for fractional QHE.

Why the term "topological phenomena"?

Topology is concerned with the connectedness of space; properties that are preserved under continuous deformations.



The number of holes is preserved

The surfaces have the same connectedness

Both have the same Euler Characteristic, χ.

$$\boxed{\int_{\substack{\text{all surface}\\ \text{of object}}} \left(\text{Gaussian Curvature}\right) dA = 2\pi \chi}$$

Gauss-Bonnet Theorem

Spheres & sausages : $\chi = 2$

Coffee cups & donuts : $\chi = 0$

A similiar integral is performed in <u>k-space</u> when calculating the Hall resistance of a 2D electron gas.

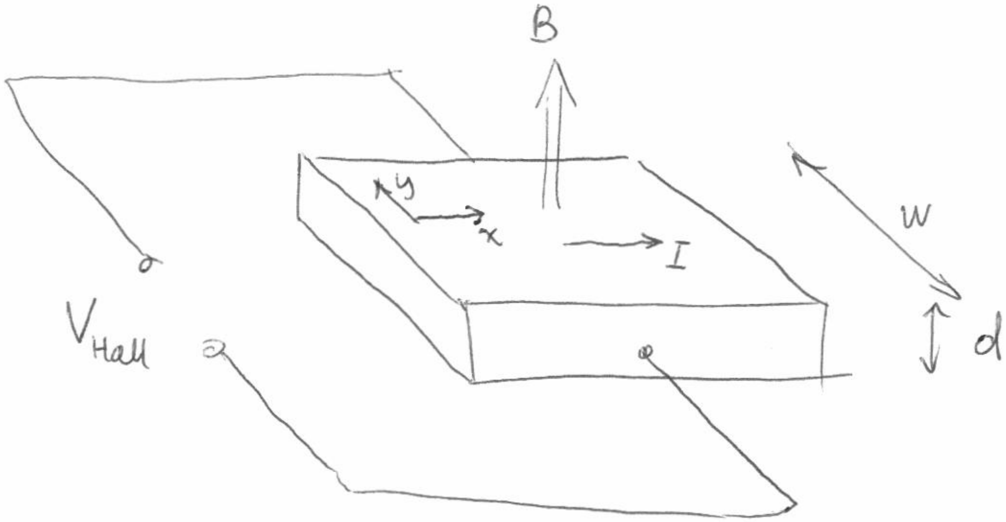
Quantized values of Hall resistance arise, even if the sample is dirty or misshapen.

Applications of topological phenomena

QHE $\longrightarrow$ R_{standard} , 11 significant figures

a.c. JE $\longrightarrow$ V_{standard} , 13 significant figures

Before tackling the QHE, consider the basics of how we describe e^- transport in a B-field.



Thin sample, height d width w.
Current source (not shown) drives I.

$\boxed{\text{EXERCISE}}$

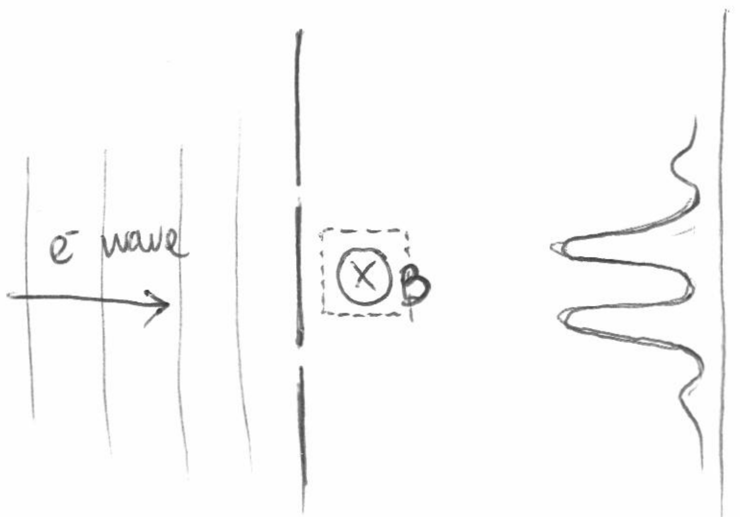
Assuming the Drude model (all e^-s drift at velocity $v = \frac{eE\tau}{m}$)

Show that $V_{Hall} = \dfrac{-IB}{n_{3d}\,e\,d} = \dfrac{-IB}{n_{2d}\,e}$

(common method to characterize n_{2d})

AHARANOV-BOHM (AB) EFFECT

[Force is not enough to explain everything that e^-'s do in a B-field]



Change the interference pattern without ever exerting a force on the e^-.

Perhaps not surprising (?) since in QM everything is formulated in terms of energy & momentum (there are no explicit "force" terms in the Hamiltonian)

How <u>do</u> we add a B-field to the Hamiltonian of a traveling e^-.

Velocity dependent potential for a moving e^-

$$U = eV(\vec{r}) - e\vec{v}\cdot\vec{A}$$

position dependent term

velocity dependent term.

Classical physics, describing Lorentz force using potentials

EXERCISE

a) Find the $\vec{B}$-field associated with $\vec{A} = -B_0 y\,\hat{x}$

b) If $V(\vec{r}) = 0$ and $\vec{v} = v_0\hat{x}$

find $\frac{\partial U}{\partial x}$, $\frac{\partial U}{\partial y}$ & $\frac{\partial U}{\partial z}$ to make sure it is consistent with Lorentz force.

Without a velocity-dep potential, $\vec{p} = m\vec{v}$ and position $\vec{r}$ are conjugate variables

i.e. $p_x = \frac{\partial L}{\partial \dot{x}}$ where $L = K.E. - P.E.$ etc.

Now $\dot{x}$ is mixed into the potential energy term we rethink how we define $\vec{p}$.

$$\vec{p} = m\vec{v} + e\vec{A}$$

"Canonical momentum" is conjugate variable to $\vec{r}$.

Using these classical conjugate variables to rewrite the S. Eq^n yields

$$\frac{1}{2m}\left(-i\hbar\nabla - e\vec{A}\right)^2\Psi + V(\vec{r})\Psi = i\hbar\frac{d\Psi}{dt}$$

Canonical momentum $(\vec{p} = m\vec{v} + e\vec{A})$ tells us how the phase of a Bloch wave state will evolve as the e^- moves thru space:

phase accumulated along a trajectory

$$\phi = \int_{Traj} \vec{k}\cdot d\vec{l}$$

$$= \int \frac{\vec{p}}{\hbar}\cdot d\vec{l}$$

$$= \phi_{B=0} + \int \frac{e\vec{A}}{\hbar}\cdot d\vec{l} \qquad (1)$$

(see Feynman Lecture 15-9)

Eq^n (1) allows us to explain the AB effect.