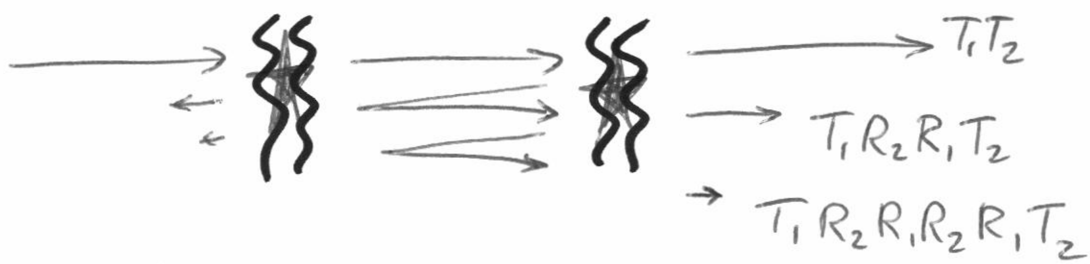
DAY 7 ① PH 671

Last time.

~~Then~~ Pair of elastic scattering sites

Next.

Pair of <u>inelastic</u> scattering sites (no interference effects)



where $T_1 = |t_1|^2 \qquad T_2 = |t_2|^2$

$\boxed{\text{EXERCISE}}$

Add together the infinite series of terms to show

$$T_{tot} = \frac{|t_1|^2 |t_2|^2}{1 - |r_1|^2 |r_2|^2}$$

Hence, the conductance of this 1d system will be

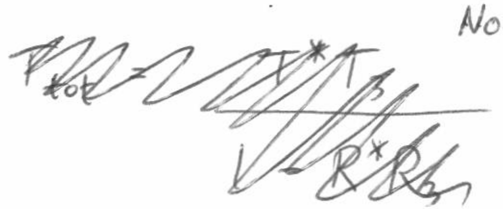
$$G = T_{tot} \frac{2e^2}{h}$$

equivalently resistance $R = \frac{1}{T_{tot}} \frac{h}{2e^2} = \frac{h}{2e^2} \frac{1 - |r_1|^2 |r_2|^2}{|t_1|^2 |t_2|^2}$

$\boxed{\text{HW \#3}}$ asks you to extend this result to many barrier in series.

Use this concept:

Described by T^*, R^* Described by T_3, R_3



Now have two barriers in series.

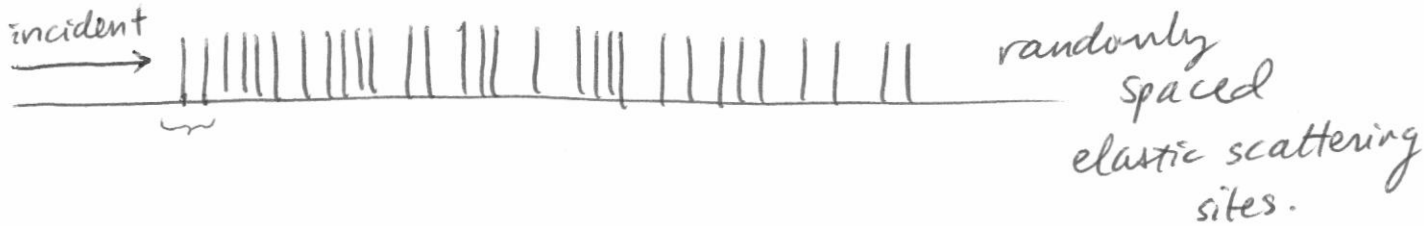
Keep adding barriers to show

$$\text{Resistance} = \frac{h}{2e^2}\left(1 + N\frac{T}{R}\right)$$

N: # of barriers

goes up linearly with the number of barriers.

Now examine the case of elastic scattering



For the first two barriers

$$T = \frac{|t_1|^2|t_2|^2}{1 + |r_1|^2|r_2|^2 - 2|r_1||r_2|\cos(\phi^*)}$$

$$\text{Resistance} = \frac{h}{2e^2}\,\frac{1 + |r_1|^2|r_2|^2 - 2|r_1||r_2|\cos(\phi^*)}{|t_1|^2|t_2|^2}$$

$$\langle \text{Resistance} \rangle = \frac{h}{2e^2}\,\frac{1 + |r_1|^2|r_2|^2}{|t_1|^2|t_2|^2} \qquad \text{Avg over all } \phi^* \quad \text{——— (i)}$$

If I lump together the first 100 barriers (Reflection $R \approx 1-\epsilon$, transmission $T = \epsilon$)

And add one more barrier (Reflection $dR \ll 1$ Transmission $1 - dR$)

And use eq^n (1)

$$\langle \text{Resistance of 101 barriers} \rangle = \frac{h}{2e^2}\,\frac{1 + R\,dR}{T(1-dR)}$$

$$\approx \frac{h}{2e^2}\,\frac{1}{T}\,(1+dR)(1+dR) \qquad 1^{st} \text{ order expansion in } dR$$

$$= \langle \text{Resistance of 100 barriers} \rangle (1 + 2dR)$$

The additional reflection dR does not add a fixed amount to the total resistance.

Instead, it adds an amount that is proportional to the original resistance.

More scattering sites, resistance rises exponentially.

$$\text{Resistance} = \frac{h}{2e^2} \exp\left(\frac{2L}{l_e}\right)$$

Note: This mathematics can be used in ~~the same~~ a similar way to analyze incoherent scattering

$$\text{Resistance of 101 barriers} = \frac{h}{2e^2} \frac{(1-RdR)}{T(1-dR)}$$

$$= \frac{h}{2e^2} \frac{1}{T} (1-RdR)(1+dR)$$

$$= \frac{h}{2e^2} \frac{1}{T} (1-RdR+dR) \quad \text{(first order terms)}$$

$$= \frac{h}{2e^2} \frac{1}{T} \left(1 + dR(1-R)\right)$$

$$= \frac{h}{2e^2} \frac{1}{T} (1 + dRT)$$

$$= \frac{h}{2e^2} \frac{1}{T} + \underbrace{\frac{h}{2e^2} dR}_{\text{adds a constant amount.}}$$

The elastic scattering length l_e is defined as $dR = \frac{dL}{l_e}$

i.e. If length of system is dL, reflection probability due to elastic scattering is $\frac{dL}{l_e}$.

A similiar defn for <u>inelastic</u> scattering

$$dR = \frac{dL}{l_i}$$ for scattering caused by inelastic processes.

Summarizing the results for many inelastic scattering events vs. many elastic scattering events

Dominate type of scattering	Length scales	Resistance
~~ki~~ inelastic	$l_i < l_e$* $l_i \ll L$	$\frac{h}{2e^2}\left(1 + \frac{L}{l_i}\right)$
elastic	$l_e < l_i$ $l_e \ll L$	$\frac{h}{2e^2}\exp\left(\frac{2L}{l_e}\right)$

Exponentially increasing resistance vs. length due to wave interference is the Hallmark of "Anderson Localization".

- Coherent waves tend to get stuck in one area if elastic scattering is strong.
- Happens both with light waves & e^- waves.

— Storzer et al. PRL <u>96</u> 063904 (2006)
— Sundqvist et al. NanoLetters (2007)

REVIEW OF WHAT WE'VE COVERED

Sommerfeld model

$\hookrightarrow \quad J = \frac{ne^2\tau}{m} E_0 \qquad (l_i < l_e)$

- Add e^--ph interaction & phonon occupation fn

  $\hookrightarrow$ Temp dependent resistance

- Consider no scattering limit $\left(\begin{matrix} L < l_i \\ L < l_e \end{matrix}\right)$

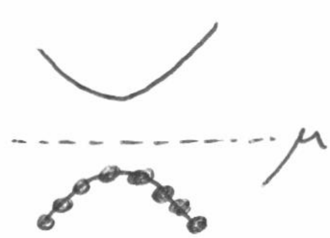
  $$\text{Resistance} = \frac{h}{2e^2} N$$ ← # of subbands

- Consider $l_e < l_i$

  $\hookrightarrow$ Anderson Localization.

Up to now
Always considering cases where μ crosses e^- states

Next: Cases where μ does not cross e^- states.

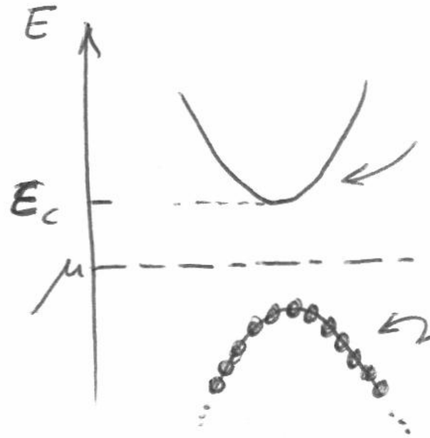


LIGHTLY DOPED SEMICONDUCTORS

Compared to metals ($n_{3d} \approx 10^{28}\ m^{-3}$)

lightly doped semics have $n_{3d} \approx 10^{20} - 10^{22}$

@ $T = 300K$.



$$n_{\substack{\text{cond}\\ \text{band}}} = \int_{E_c}^{\infty} DOS(E)\, f(E)\, dE$$

Fermi-Dirac fn.

Valance band is filled (except for a few holes) can't carry current, so we don't count these e^-s.

$$n_{\substack{\text{cond}\\ \text{band}}} = \int_{E_c}^{\infty} \frac{m_{eff}^{3/2}}{\hbar^3 \pi^2} (E - \varepsilon_c)^{1/2} \underbrace{\frac{1}{e^{(E-\mu)/k_BT} + 1}}_{\text{Fermi-Dirac fn}} dE$$

when $\varepsilon_c - \mu \gg k_BT$

$$\approx \text{const} \exp\left(- \frac{(\varepsilon_c - \mu)}{k_BT} \right) \int_0^{\infty} E^{1/2} e^{-E/kT} dE$$

$$= \text{const} \exp\left(- \frac{(\varepsilon_c - \mu)}{k_BT} \right) (k_BT)^{3/2}$$

n is very sensitive to T.