Last time

Initial e^- wave fn

Final e^- wave fn

$$\text{Probability} = \frac{2\pi}{\hbar} \underbrace{\left| \langle \vec{k} | H' | \vec{k}' \rangle \right|^2}_{\text{matrix element for transition}} DOS(\vec{k})\, d\vec{k}'\, dt$$

~~Last time~~

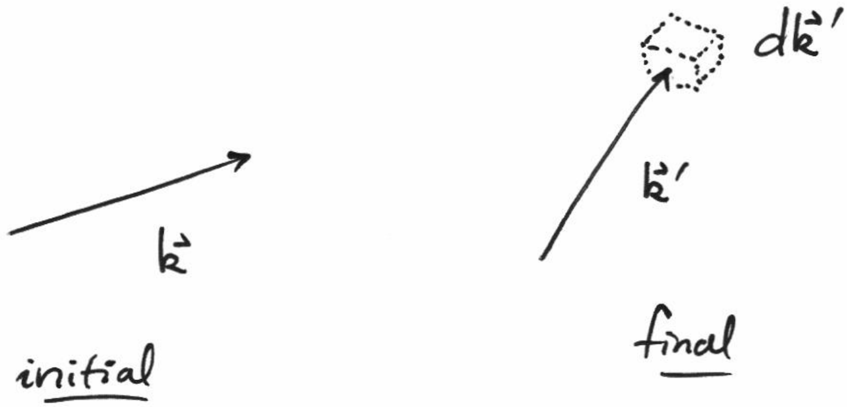
$$\langle \vec{k} | H' | \vec{k}' \rangle = \int u^*_{\vec{k}}(\vec{r})\, \vec{u}_{k_{ph}} \cdot \frac{dV}{d\vec{r}}\, u_{\vec{k}'}(\vec{r})\, d\vec{r}$$

if energy & momentum are conserved

$$\left| \langle \vec{k} | H' | \vec{k}' \rangle \right|^2 = n_{phonon} \underbrace{\left| g_{\vec{k}\vec{k}'} \right|^2}_{\text{electron phonon coupling constant for a given } \vec{k} \,\&\, \vec{k}' \ldots \text{ a material property.}}$$

Since $E \propto n_{phonon}$
$\propto (\text{Displacement})^2$
Depends on T.

Put the matrix element back into Fermi's Golden Rule:



$$\text{Prob} = \frac{2\pi}{\hbar} \left| \langle \vec{k} | H' | \vec{k}' \rangle \right|^2 \, DOS(\vec{k}') \, d\vec{k}' \, dt$$

$$= \frac{2\pi}{\hbar} \frac{1}{e^{\hbar\omega_{ph}/k_B T} - 1} \left| g_{\vec{k}\vec{k}'} \right|^2 \, DOS(\vec{k}') \, d\vec{k}' \, dt$$

You can see why the single relaxation time approximation often saves time!

As long as energy and momentum are conserved.

Some physical insights from this formula:

$\boxed{\text{If} \quad k_B T \gg \hbar\omega_{ph}}$ (for all ω_{ph})

$$\text{Prob} \propto \frac{1}{e^{\hbar\omega_{ph}/k_B T} - 1} \approx \frac{1}{1 + \frac{\hbar\omega_{ph}}{k_B T} - 1} = \frac{k_B T}{\hbar\omega_{ph}}$$

For all phonon modes

Resistance $\propto T$.

$\boxed{\text{If } k_B T < \hbar\omega_{ph}}$

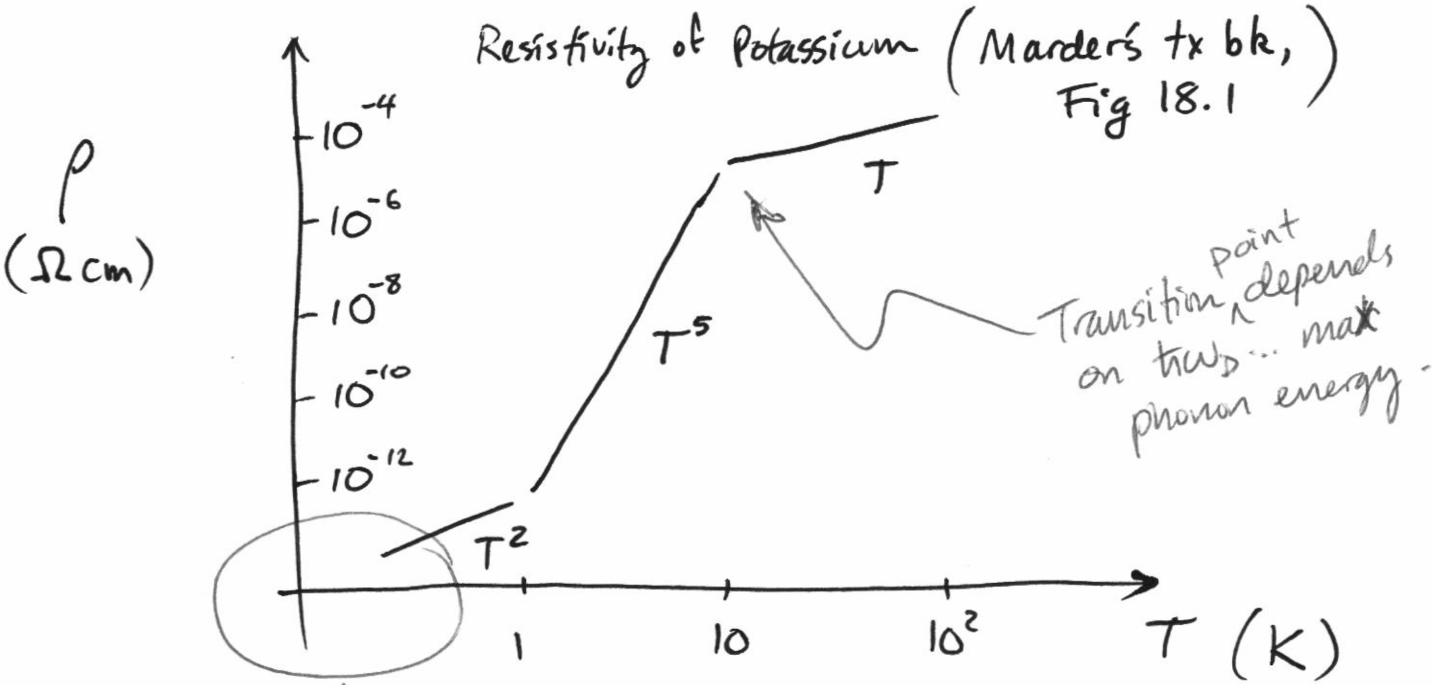
$$\frac{1}{e^{\hbar\omega_{ph}/k_BT}-1} \longrightarrow 0$$

for some ω_{ph}.

~~Some~~ Scattering events are supressed for large ω_{ph} but not small ω_{ph}.

Resistance is <u>not</u> proportional to T.

Resistivity of Potassium (Marder's tx bk, Fig 18.1)



At very low temperatures, other mechanisms will start to dominate

The max value of $\hbar\omega_{ph}$ is a very important energy in e^- transport theory.

EXERCISE: Estimate $[\hbar\omega_{ph}]_{max}$ if

$K \approx 20\,N/m$ $\qquad$ $m \approx 20\,m_p$

(show the highest freq normal mode on the ~~Phet~~ animation/simulation).

The temperature dependence of resistivity is our first concrete example where the QM treatment of e^- transport takes us beyond the Drude model.

Rest of ~~cou~~ PH671: Many more examples where the QM description predicts/explains new phenomena.

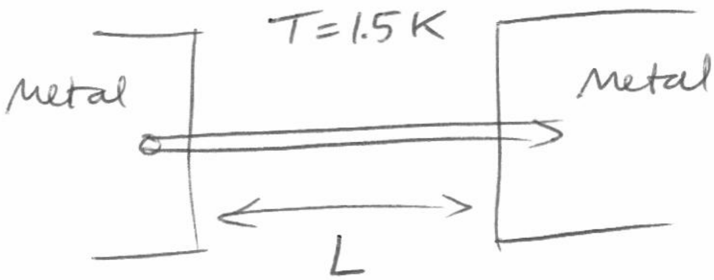
$\boxed{\text{CASE 1}}$ Cool down a perfect lattice to $T=0$

No scattering.

Assume a normal metal (not superconducting)

Is there any remaining "resistance"?

This expt has be done with CNTs



Liang et al.
Nature $\underline{411}$ 665
(2001)

$$I = \frac{4e^2}{h} V$$

$$= \frac{V}{6.5\, k\Omega}$$

Independent of L.

There is nothing to cause scattering.

Apply QM description of e^- transport to this system.