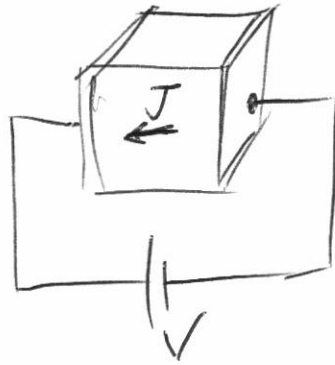


Last time

We set up the integral to solve $J(V)$



↑
current density

Before you do the integral,
let's find the scaling relation assuming

J depends on n, τ, m, eE_0, e

↑
(Force)
assume free e^- model

Exercise: Find a dimensionally correct scaling relationship

$$J = \text{const} \cdot \textcircled{?}$$

ANS

$$J \sim \frac{ne^2\tau}{m} E_0$$

Surprisingly, the HW integral will show that this constant = 1

(2)

This Drude ~~model~~ ^{result}, $J = \frac{ne^2\tau}{m} E_0$,
arose many years before the Sommerfeld
model.

Even with the wrong microscopic picture,
Drude got the right formula.

[CONCLUSION: Dimensional analysis is a handy tool!]

BEYOND THE DRUDE RESULT

$f(\vec{k})$ might vary as a function of \vec{r} & t .

eg. Time varying \vec{E} field
Temperature gradient
etc.

$$f = f(\vec{k}, \vec{r}, t)$$

BOLTZMAN TRANSPORT EQN

$$\frac{\partial f}{\partial t} = -\vec{v} \cdot \frac{\partial f}{\partial \vec{r}} - \vec{F} \cdot \frac{1}{\hbar} \frac{\partial f}{\partial \vec{k}} + \left(\frac{\partial f}{\partial t} \right)_{\text{scatter}}$$

At steady
state this
will be zero.

Some states
vacated because
 e^- drifts out
of a local region
where no e^- are
arriving.

states vacated
if force points
in same direction
as $\frac{\partial f}{\partial \vec{k}}$.

(3)

I'm still looking for a good HW question to give practice using Boltzmann transport eqn.

The eqn puts classical e- transport on a solid mathematical footing. In principle we can compute any complicated situation.

CALCULATING SCATTERING RATES



Probability that this will occur in time dt ?

Such problems are approached using Fermi's Golden Rule,

$$\text{Probability} = \frac{2\pi}{\hbar} \left| \langle \vec{k} | H' | \vec{k}' \rangle \right|^2 \text{DOS}(\vec{k}') d\vec{k}' dt$$

↑ Perturbing Hamiltonian

See Griffiths Ch on time-dep perturb theory.

(4)

At $T=0$ in a perfect ionic lattice, $H'=0$.

e^- s are in their eigenstates, no driving force to change

At $T>0$, $H'\neq 0$ because ionic lattice is vibrating. What used to be a perfect e^- eigenstate is now perturbed.

Some Matrix elements $\langle \vec{k} | H' | \vec{k}' \rangle$ are now non-zero.



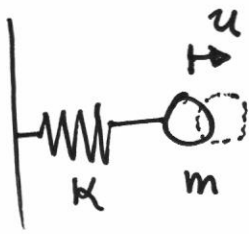
Calculate this for a vibrating lattice.

First, review the physics of lattice vibrations, which is taught in PH575.

EXERCISE: Repeat the pop quiz from PH575.

⑤

PHONONS IN A 1-ATOM SYSTEM

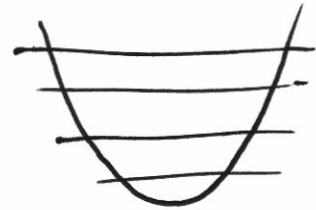


$$\omega = \sqrt{\frac{K}{m}}$$

There is only one mode.

$$u(t) = u_0 \sin \omega t$$

Energy stored in the mode



$$E = \hbar \omega (n_{ph} + 1/2)$$

Quantum # = Phonon occupation number.

$n_{ph} = 0$, no phonons in the mode

$n_{ph} = 1$, one phonon in the mode

$n_{ph} = 2$, two phonons in the mode

We expect $u_0 \propto \sqrt{E} \propto \sqrt{n_{ph}}$

since classically $\frac{1}{2} K u_0^2 = E$

Finally, the occupation # at thermal eqb $n_{ph} = f_{BE} = \frac{1}{e^{\hbar \omega / kT} - 1}$

⑥

PHONONS IN AN N-ATOM SYSTEM

There are N -modes

Each mode has a different wavevector \vec{k}
and freq $\omega_{\vec{k}}$

For \vec{k}^{th} mode

$$n_{\text{ph}, \vec{k}} = \frac{1}{e^{\hbar\omega_{\vec{k}}/kT} - 1}$$

The displacement of atoms depends on which lattice site you look at:



$$\vec{u}_{\text{ph}}(\vec{r}, t) = \text{Re} \left\{ \vec{u}_0 e^{i\vec{k}_{\text{ph}} \cdot \vec{r}} e^{-i\omega_{\vec{k}} t} \right\}$$

Now calculate $\langle \vec{k} | H' | \vec{k}' \rangle$ at $T > 0$

$|\vec{k}\rangle, |\vec{k}'\rangle$ are e^- eigenstates. H' is caused by phonons.