

Day 1

①

(Total of 14 classes)

PH 671

- Website tour

HW postings

HW policy • Drop one

• Not graded after answers posted.

Final exam • 2 hr exam

• Cheat sheet is allowed / encouraged.

Quote from Chpt 1 A&M, p 2.

"During the last hundred years..."

↖
The struggle to understand solid state systems is related to the complexity:

10^{23} electrons interacting with each other and a lattice of nuclei.

Rather than one unified theory, different models are appropriate in different situations.

A&M works through these models in a wonderfully logical sequence.

Out line :

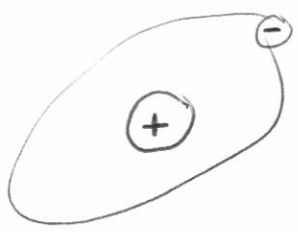
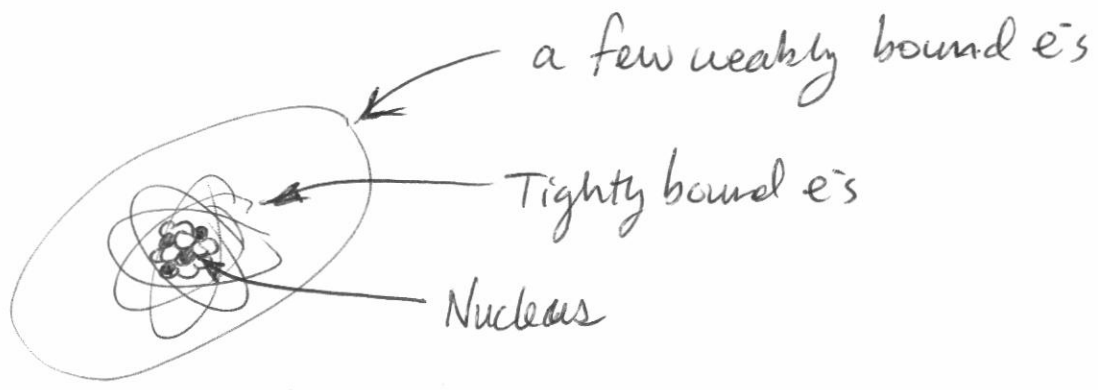
- ① Non-interacting electron gas model for metals
- ② Boltzman transport eqⁿ
- ③ Quantum transport
- ④ Topological phenomena in quantum transport
- ⑤ Tunneling phenomena
- ⑥ Superconductivity

The Sommerfeld Theory of Metals

A&M Chpt 2

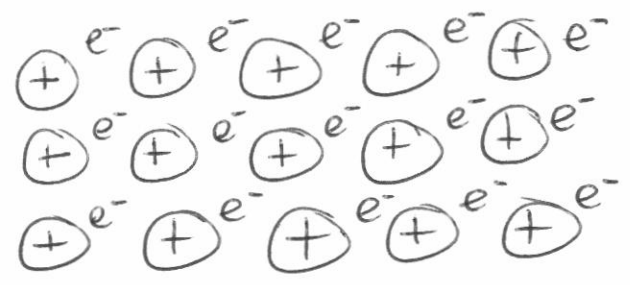
The PhD supervisor
of Heisenberg
Pauli
Debye
& Bethe

All went on to win Nobel prizes.



A weakly bound e^- (1 or more).

Ion core, for example 19 protons, 18 e^- s



Lattice of ions.
Each contributes
1 or more
delocalized e^- s.

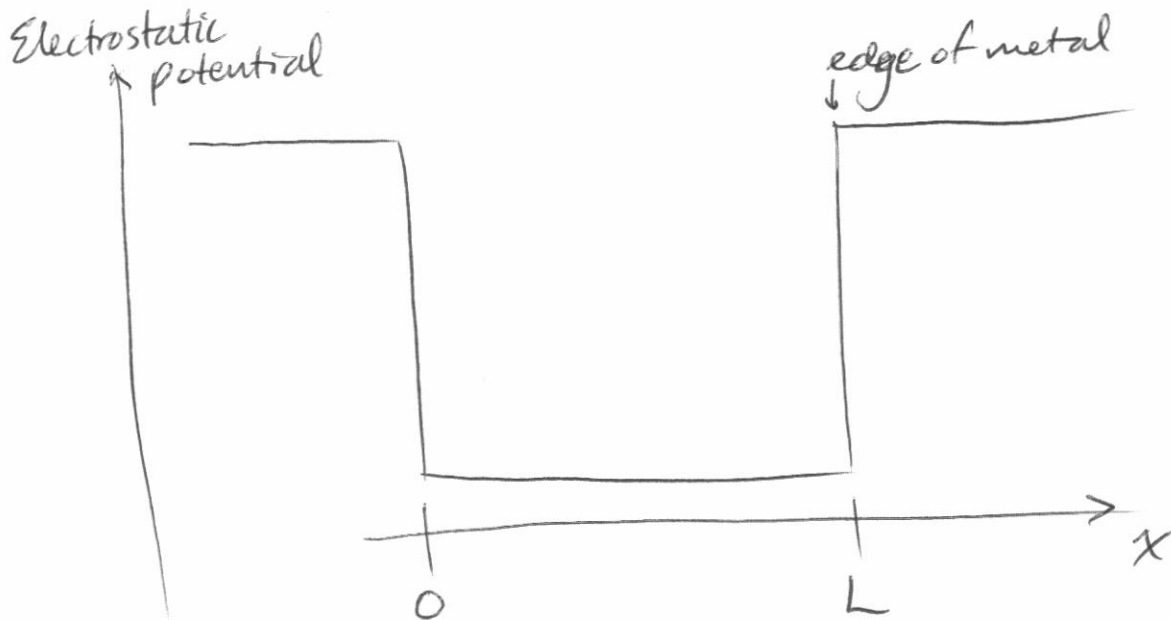
(4)

The density of e^- s is phenomenal.

$$n \approx \frac{1}{(0.3 \text{ nm})^3} = \frac{1}{0.1 \times 10^{-28}}$$

$$n = \frac{1}{(3 \text{ \AA})^3} = \frac{1}{27 \times 10^{-30} \text{ m}^3} = \frac{1}{0.3 \times 10^{-28}} \\ = 3 \times 10^{28} \text{ m}^{-3}$$

The ion lattice keeps these e^- s packed tightly together
→ Dense gas of e^- s.



e^- s stay inside the ion lattice
~~metal~~, lower potential energy.

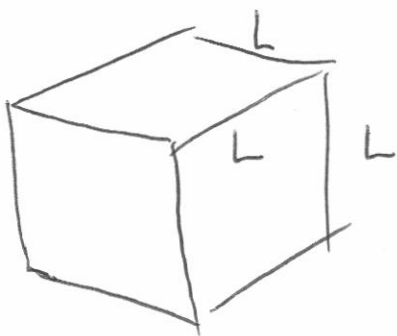
(5)

To complete the Sommerfeld model we assume

- The ion lattice fills a volume V
- Electrons don't interact with each other.

And we ask:

Which quantum states will be occupied?



$$L^3 = V$$

Ground state $\psi_0 = \begin{cases} A \sin \frac{\pi x}{L} \sin \frac{\pi y}{L} \sin \frac{\pi z}{L} & \text{inside} \\ 0 & \text{outside} \end{cases}$

[I've assumed an infinite potential well in 3d]

Other allowed states

$$\psi(n_x, n_y, n_z) = A \sin \frac{n_x \pi x}{L} \sin \frac{n_y \pi y}{L} \sin \frac{n_z \pi z}{L}$$

where n_x , n_y & n_z are positive integers.

⑥

I have a lot of e^- s to add to the system.

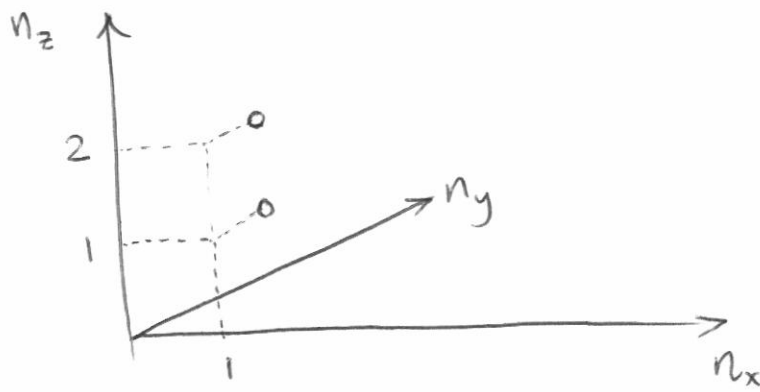
$$nV$$

First e^- goes in the ground state

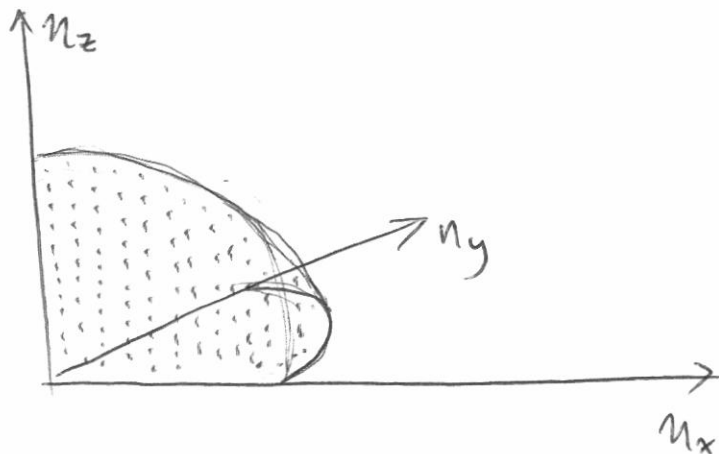
n_x	n_y	n_z
1	1	1

The second e^- goes ?

The third e^- goes ?



Convenient to imagine these states on a 3d grid.



(7)

EXERCISE: Calculate the K.E., $\frac{p^2}{2m}$, of an arbitrary state in this system.

ANSWER:
$$\frac{\hbar^2 \pi^2}{2mL^2} (n_x^2 + n_y^2 + n_z^2)$$

This is the length squared of a vector on the 3d grid.

States that have equal energy are equidistant from the origin of the 3d grid.

As more states are occupied, the length $\sqrt{n_x^2 + n_y^2 + n_z^2}$ grows bigger.

When $\sqrt{n_x^2 + n_y^2 + n_z^2}$ reaches n_{\max} we have

$$nV = \frac{2 \cdot \frac{4}{3} \pi n_{\max}^3}{8} = \frac{\pi}{3} n_{\max}^3$$

(8)

n_{\max} is related to the kinetic energy of the last occupied states

$$E_{\max} = \frac{\hbar^2 \pi^2}{2mL^2} n_{\max}^2$$

(E_{Fermi})

$$= \frac{\hbar^2 \pi^2}{2mL^2} \left(\frac{3nL^3}{\pi} \right)^{2/3}$$

$$= \frac{\hbar^2}{2m} \left(3\pi^2 n \right)^{2/3}$$

concentration
of free electrons
in the cube.

The length factors cancel:

Larger systems have lower energy wavefns,
but they also have more e^- s to fill the wavefns.

E_{\max} is officially called E_{Fermi}