

PH632
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HW #8

Instructor
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① Silver
Lining

$$\tilde{k}^2 = \epsilon \mu \omega^2 + i \mu \sigma \omega$$

$$\sigma = 3 \times 10^7 \quad \omega = 2\pi \times 10^{10}$$

Assume $\epsilon \rightarrow \epsilon_0$ and $\mu \rightarrow \mu_0$

$$\operatorname{Re}\{\tilde{k}^2\} = \frac{(2\pi \times 10^{10})^2}{(3 \times 10^8)^2}$$

$$\approx 4 \times 10^{20} \times 10^{-16}$$

$$= 4 \times 10^4$$

$$\operatorname{Im}\{\tilde{k}^2\} = 4\pi \times 10^{-7} \times 3 \times 10^7 \times 2\pi \times 10^{10}$$

$$= 24\pi^2 \times 10^{10}$$

$$\approx 240 \times 10^{10}$$

$$\operatorname{Im}\{\tilde{k}^2\} \gg \operatorname{Re}\{\tilde{k}^2\}$$

Therefore

$$\tilde{k} \approx \sqrt{240} \frac{1}{\sqrt{2}} (1+i) \times 10^5$$

$$\approx 10^6 (1+i)$$

$$\delta = \frac{1}{\operatorname{Im}\{\tilde{k}\}} = \frac{1}{10^6} = 1 \mu\text{m}.$$

A coating of $5 \mu\text{m}$ would work well.

The fraction of microwave intensity leaking out would be $(e^{-5})^2 = 5 \times 10^{-5}$

(less than 1 part in 10,000).

Physics pages

Notes on topics in science

Skin depth of water and metals

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education – Chapter 9, Post 19a-b.

Electromagnetic waves in a conductor (where there is free current but no free charge) can be written as

$$\tilde{\mathbf{E}}(z, t) = \tilde{\mathbf{E}}_0 e^{i(\tilde{k}z - \omega t)} \quad (1)$$

$$\tilde{\mathbf{B}}(z, t) = \tilde{\mathbf{B}}_0 e^{i(\tilde{k}z - \omega t)} \quad (2)$$

where the wave vector is complex:

$$\tilde{k} = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}} \sqrt{\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} + 1} + i \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}} \sqrt{\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1} \equiv k + i\kappa \quad (3)$$

For a poor conductor, the conductivity σ is small, so for large enough frequencies $\sigma \ll \epsilon\omega$ and we can approximate κ by

$$\kappa \approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}} \sqrt{1 + \frac{1}{2} \left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1 \quad (4)$$

$$= \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} \quad (5)$$

Since the imaginary part of \tilde{k} governs the attenuation of the wave as it penetrates the material, the skin depth for a poor conductor is

$$d = \frac{1}{\kappa} = \frac{2}{\sigma} \sqrt{\frac{\epsilon}{\mu}} \quad (6)$$

For pure (deionized) water $\sigma = 5.5 \times 10^{-6} \text{ S m}^{-1}$ and $\epsilon = 80.1\epsilon_0$ (at 20°C) (we can take $\mu \approx \mu_0$) so the skin depth of water is

$$d = 8635 \text{ m} \quad (7)$$

Because the skin depth is so large, water is transparent.

For a good conductor, $\sigma \gg \epsilon\omega$ and we can approximate

$$\kappa \approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}} \sqrt{\frac{\sigma}{\epsilon\omega}} = \sqrt{\frac{\mu\sigma\omega}{2}} \approx k \quad (8)$$

so the skin depth is

$$d = \sqrt{\frac{2}{\mu\sigma\omega}} \approx \frac{1}{k} = \frac{2\pi}{\lambda} \quad (9)$$

where λ is the wavelength within the material. For a typical metal, $\sigma \approx 10^7 \text{ S m}^{-1}$ and $\mu \approx \mu_0$ so the skin depth at visible frequencies $\omega \approx 10^{15} \text{ s}^{-1}$ is

$$d \approx 1.26 \times 10^{-8} \text{ m} \quad (10)$$

With a skin depth this small, even a thin film of metal is effectively impervious to any penetration by visible light.

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