PH632 Winter 2016

Electromagnetic Theory II

Problem Set #8 Due: Friday 3-11-2016

1. Silver lining

Silver is an excellent conductor, but it is expensive. Suppose you were designing a microwave experiment to operate at a frequency of $f = 10^{10}$ Hz. How thick would you make the silver coatings?

Note: The conductivity of silver at microwave frequencies is $3 \times 10^7\ \Omega^{-1}\text{m}^{-1}$.

2. Good conductors, bad conductors

a) Show that the skin depth in a poor conductor ($\sigma \ll \omega\epsilon$) is $(2/\sigma)\sqrt{\epsilon/\mu}$ (independent of frequency).

b) Show that the skin depth for in a good conductor ($\sigma \gg \omega\epsilon$) is $\lambda/2\pi$ (where λ is the wavelength *in the conductor*). Find the skin depth (in nanometers) for a typical metal ($\sigma \approx 10^7\ \Omega^{-1}\text{m}^{-1}$) in the visible range ($\omega \approx 10^{15}\ \text{s}^{-1}$) assuming $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$. Why are metals opaque?

c) Show that in a good conductor the magnetic field lags the electric field by 45 deg, and find the ratio of their amplitudes. For a numerical example, use the typical metal in part b.

3. Intensity inside a conductor

A plane wave, traveling in the x-direction, hits a metal ($\sigma \gg \omega\epsilon$) at normal incidence.

a) Show that the time-averaged energy density of the plane wave inside the metal is

$$\frac{\left(\text{Re}\{\tilde{k}\}\right)^2}{2\mu\omega^2} E_0^2 e^{-2(\text{Im}\{\tilde{k}\})x}$$

b) Show that the intensity of the plane wave inside a conductor is

$$\frac{\text{Re}\{\tilde{k}\}}{2\mu\omega} E_0^2 e^{-2(\text{Im}\{\tilde{k}\})x}$$

Note: You can adopt Griffith's notation $\text{Re}\{\tilde{k}\} = k$ and $\text{Im}\{\tilde{k}\} = \kappa$.