Electromagnetic Theory II

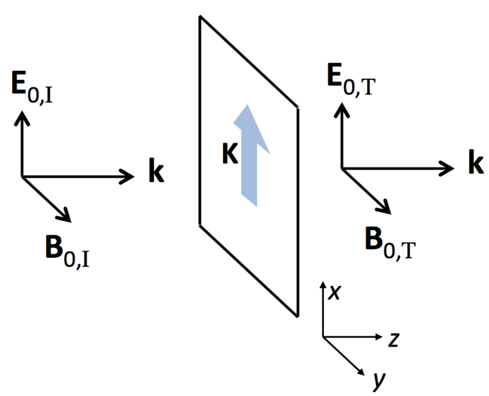
Problem Set #7 Due: Friday 3-4-2016

1. Light of angular frequency ω passes from medium 1, through a slab (thickness *d*) of medium 2, and into medium 3 (for instance, from water though glass into air). Show that the combined transmission coefficient for normal incidence is given by

$$T^{-1} = \frac{1}{4n_1n_3}\left[(n_1+n_3)^2 + \frac{(n_1^2-n_2^2)(n_3^2-n_2^2)}{n_2^2}\sin^2\left(\frac{n_2\omega d}{c}\right)\right]$$

Hint: To the *left*, there is an incident wave and a reflected wave; to the *right* there is a transmitted wave; inside the slab, there is a wave going to the right and a wave going to the left. Express each of these in terms of its complex amplitude, and relate the amplitudes by imposing suitable boundary conditions at the two interfaces. All three media are linear and homogeneous; assume $\mu_1 = \mu_2 = \mu_3 = \mu_0$.

2. Linearly polarized monochromatic light hits a sheet of graphene at normal incidence. The graphene sheet is suspended so there is air on both sides of the graphene. The incident E-field, $\mathbf{E}_{0,I}$, drives a sheet current in the graphene, $\mathbf{K} = \sigma\mathbf{E}$ where $\sigma = (\pi/4)\times 2e^2/h$. The diagram below shows incident and transmitted light (you also need to consider reflected light).



(a) What boundary condition must be satisfied by $\mathbf{B}(z = 0^-)$ and $\mathbf{B}(z = 0^+)$?

(b) Find the fraction of light is transmitted? Express your answer as a percentage.

3. The Lorentz Oscillator Model

Consider a gas of atoms made up of *N* atoms per unit volume. We model each atom as an electron on a spring so that the refractive index of the material is given by

$$n \cong 1 + \frac{Ne^2}{2m\epsilon_0} \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$$

where ω_0 is the resonant frequency of the electron on a spring and γ is the damping coefficient.

a) Use the equation above to calculate the group velocity of electromagnetic plane waves in the gas.

b) Use a computer to graph the group velocity as a function of $(\omega/\omega_0)^2$ from 0 to 2. Let $\frac{Ne^2}{2m\epsilon_0\omega_0^2} = 0.003$ and $\gamma = (0.1)\omega_0$.