Electromagnetic Theory II

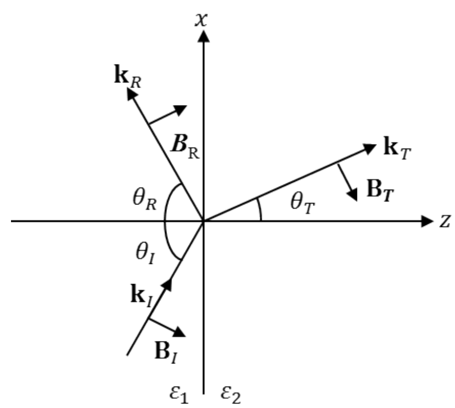
Home Work #6 Due: 2-26-2014

1. The real electric field for a monochromatic plane wave of amplitude E_0, frequency ω, phase angle $\phi = 0$, wave vector $\mathbf{k}$ and polarization $\boldsymbol{\epsilon}$ is expressed as

$$\mathbf{E}(\mathbf{x}, t) = \boldsymbol{\epsilon}\, E_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$

Write down the real *electric* and *magnetic* fields that is (a) traveling in the negative x direction and polarized in the z direction; (b) traveling in the direction from the origin to the point $(1,1,1)$, with polarization parallel to the xz plane. In each case, sketch the wave, and give the explicit Cartesian components of $\mathbf{k}$ and $\boldsymbol{\epsilon}$.

2. Analyze the refraction and reflection of s-polarization electromagnetic waves at a plane interface as shown in the figure below.



(a) Impose the boundary conditions and obtain the Fresnel equations for E_{0T} and E_{0R}:

$$E_{OT} = \left(\frac{2}{1 + \alpha\beta}\right) E_{OI}$$

$$E_{OR} = \left(\frac{1 - \alpha\beta}{1 + \alpha\beta}\right) E_{OI}$$

where $\alpha = \cos\theta_T/\cos\theta_I$ and $\beta = \mu_1 n_2/\mu_2 n_1$.

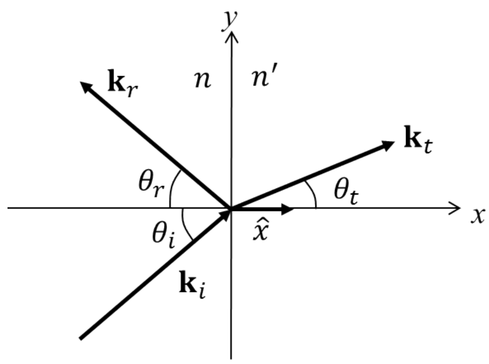
(b) Sketch E_{0R}/E_{0I} and E_{0T}/E_{0I} as functions of θ_I, for the case $\beta = \frac{n_2}{n_1} = 1.5$.

(c) Show that there is no Brewster's angle for any n_1 and n_2.

(d) Confirm that the Fresnel equations reduce to the proper forms at normal incidence.

(e) Compute the reflection and transmission coefficients, and check that they add up to 1.

3. Prove that the energy flux is conserved for an electromagnetic wave incident on a planar boundary for both s- and p-polarizations (see the figure below): $\hat{x} \cdot (\mathbf{S_i} + \mathbf{S_r}) = \hat{x} \cdot \mathbf{S_t}$.



Note: *Definition of s and p polarization*

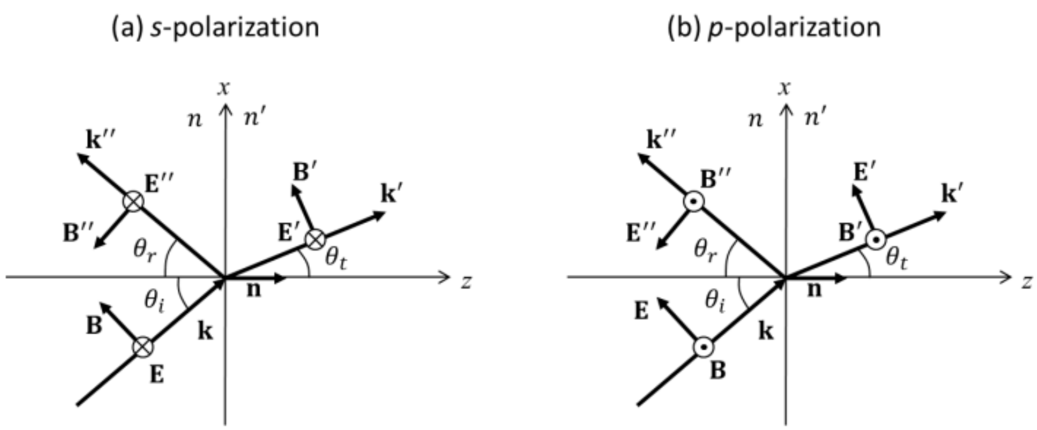


Fig 7.7 Reflection and refraction with polarization (a) perpendicular (s-polarization) and (b) parallel (p-polarization) to the plane of incidence