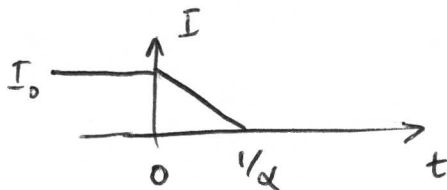
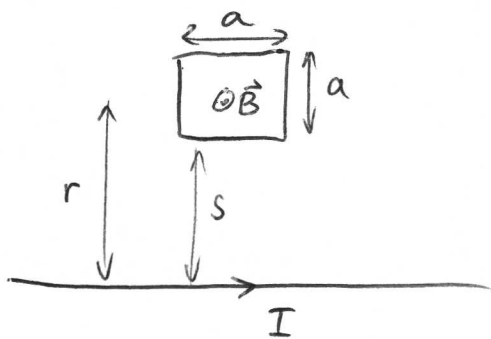


HW #5

①



$$B = \frac{\mu_0 I}{2\pi r}$$

$$\begin{aligned} \Phi &= a \int_s^{s+a} \frac{\mu_0 I}{2\pi r} dr \\ &= \frac{a \mu_0 I}{2\pi} \ln\left(\frac{s+a}{s}\right) \end{aligned}$$

When the current is turned off,

$$\text{EMF} = \frac{d\Phi}{dt} = \frac{a \mu_0}{2\pi} \alpha I_0 \ln\left(\frac{s+a}{s}\right) \quad 0 < t < \frac{1}{\alpha}$$

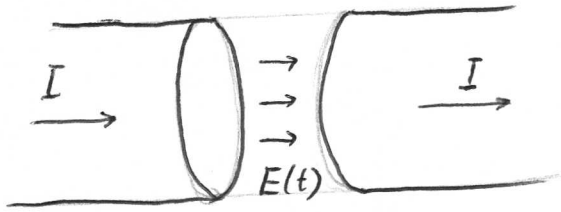
The current in the small loop

$$i = \frac{\text{EMF}}{R} = \frac{a \mu_0}{2\pi R} \alpha I_0 \ln\left(\frac{s+a}{s}\right) \quad 0 < t < \frac{1}{\alpha}$$

Total charge flowing past a point

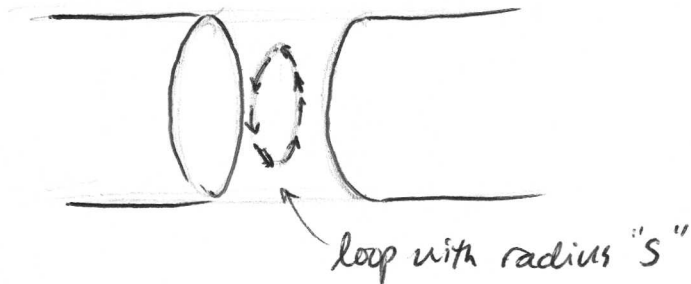
$$\int_0^{1/\alpha} i dt = \frac{a \mu_0 I_0}{2\pi R} \ln\left(\frac{s+a}{s}\right)$$

(2)



$$E(t) = \frac{\sigma(t)}{\epsilon_0} = \frac{I t}{\pi a^2 \epsilon_0}$$

$$\frac{dE}{dt} = \frac{I}{\pi a^2 \epsilon_0} \Rightarrow J_d = \frac{I}{\pi a^2}$$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int J_d da$$

$$2\pi s B = \frac{\mu_0 I}{\pi a^2} \pi s^2$$

$$\boxed{\vec{B} = \frac{\mu_0 I s}{2\pi a^2} \hat{\phi}}$$

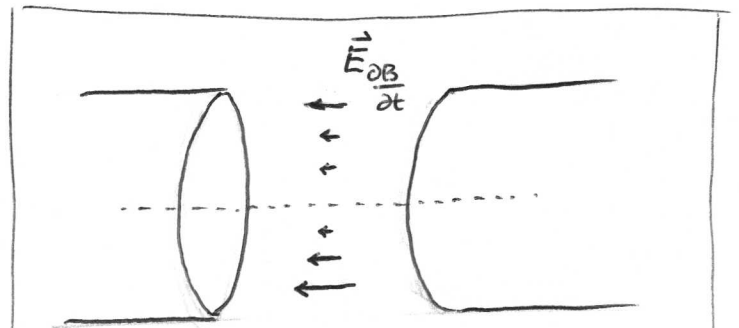
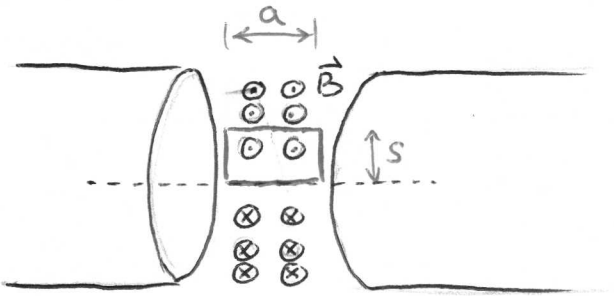
(3)

BONUSAt low freq $E_{rf} = E_0 \cos \omega t$

$$\frac{dE}{dt} = -\omega E_0 \sin \omega t$$

$$J_D = -\epsilon_0 \omega E_0 \sin \omega t$$

$$\Rightarrow \vec{B} = \frac{-\mu_0 \epsilon_0 \omega E_0 \sin \omega t}{2} s \hat{\phi}$$



$$\nabla \times \vec{E}_{\frac{\partial B}{\partial t}} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E}_{\frac{\partial B}{\partial t}} \cdot d\vec{\ell} = -\frac{d}{dt} \int_{\text{area}} \vec{B} \cdot d\vec{a}$$

Make a guess that

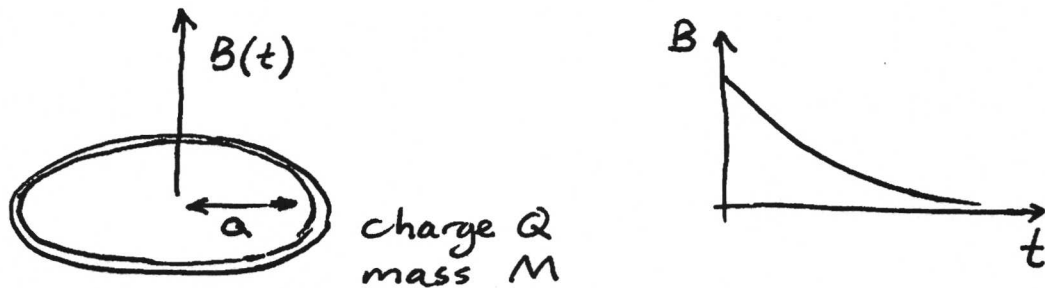
$\vec{E}_{\frac{\partial B}{\partial t}}$ points along wire axis and is zero on center line (because B is zero on center line).

$$a E_{\frac{\partial B}{\partial t}}(s) = -\frac{d}{dt} a \int_0^s \frac{-\mu_0 \epsilon_0 \omega E_0 \sin \omega t}{2} s \, ds$$

$$= \epsilon_0 \mu_0 \frac{a}{2} \omega^2 E_0 \cos \omega t \frac{s^2}{2}$$

$$E_{\frac{\partial B}{\partial t}} = \frac{\omega^2 s^2}{4c^2} E_0 \cos \omega t$$

(4)



Changing B -field causes a voltage around the ring

$$\int \vec{E} \cdot d\vec{l} = \text{EMF} = \pi a^2 \frac{dB}{dt}$$

$$E \cdot 2\pi a = \pi a^2 \frac{dB}{dt}$$

$$E = \frac{a}{2} \frac{dB}{dt}$$

(direction can be figured out from Lenz's Law, $\hat{\phi}$ dir)

The torque on the ring is

$$\tau = (\text{Force}) \times (\text{Lever arm})$$

$$= EQa$$

$$= \frac{a^2 Q}{2} \frac{dB}{dt} = \frac{a^2 Q}{2} \alpha B_0 e^{-\alpha t} \hat{z}$$

The change in angular momentum

$$\Delta L = \int_0^\infty \tau dt = \int_0^\infty \frac{a^2 Q}{2} \frac{dB}{dt} dt$$

$$= \frac{a^2 Q}{2} \int_0^\infty dB$$

$$= \frac{a^2 Q B_0}{2}$$

The final L does not depend on α .

If B decays slowly there is less force, but it lasts longer.