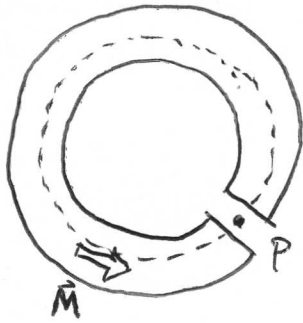


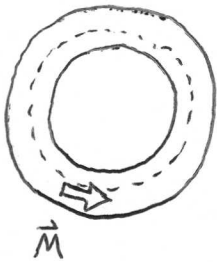
# Homework #4



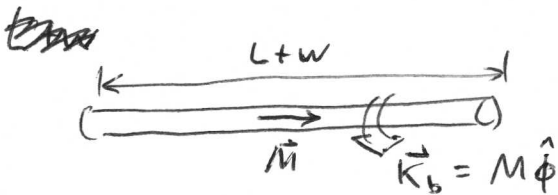
Dashed line is length  $L+w$

Find  $B$  at point  $P$ .

Model the system as two pieces and then superimpose

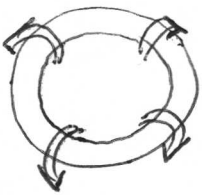


Piece 1: A complete torus made from a rod of length  $L+w$ .



The total <sup>bound</sup> current circulating around the cylinder is

$$I = (L+w)M.$$



The total <sup>bound</sup> current circulating around the torus is  $(L+w)M$ .

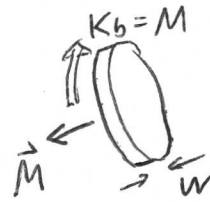
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (L+w)M$$

center line of rod

$$\Rightarrow B = \mu_0 M$$



Piece 2: A disk.



Total <sup>bound</sup> current circulating around disk is

$$I = K_b w = Mw$$

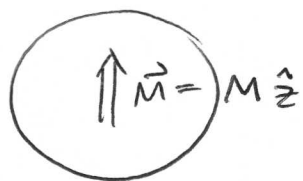
$$B \approx \frac{\mu_0 I}{2} \frac{a^2}{(a^2+z^2)^{3/2}} = \frac{\mu_0 I}{2a}$$

$$= \mu_0 M \frac{w}{2a}$$

Super imposing Piece 1 & Piece 2

$$B_{tot} = \mu_0 M \left(1 - \frac{w}{2a}\right)$$

(2)



Bound sheet current

$$\vec{K}_b = M \sin \theta \hat{\phi}$$

compare to a <sup>spinning</sup> hollow sphere that has surface charge density  $\sigma$ .



$$\vec{v} = \omega R \sin \theta \hat{\phi}$$

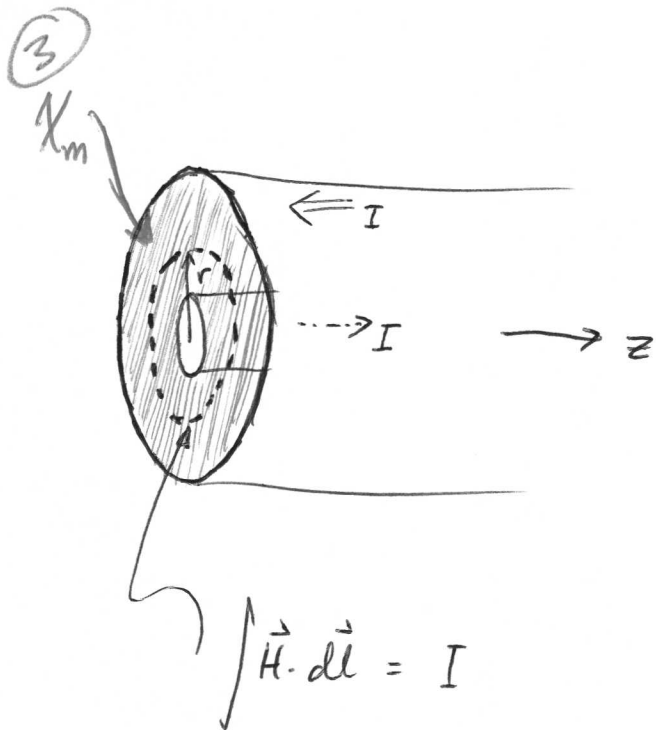
$$\vec{K} = \sigma \vec{v} = \sigma \omega R \sin \theta \hat{\phi}$$

The spinning hollow sphere will produce the same  $\vec{B}$  field as the magnetized sphere when

$$M = \sigma \omega R$$

$$\Rightarrow \vec{B}_{\text{inside}} = \frac{2}{3} \mu_0 \vec{M}$$

$$\vec{B}_{\text{outside}} = \frac{\mu_0 M R^3}{3 r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$$



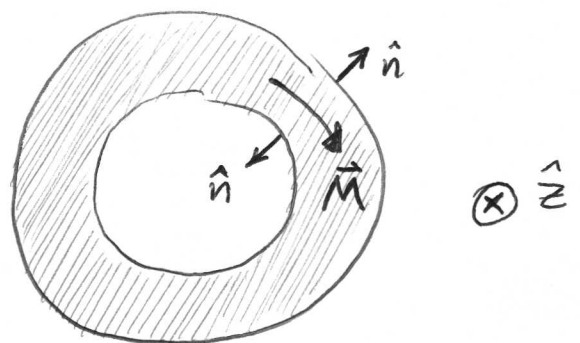
$$\vec{H} = \frac{I}{2\pi r} \hat{\phi} \quad \text{for } a < r < b.$$

Linear material so  $\vec{B} = \mu_0(1 + \chi_m)\vec{H}$

$$\vec{B} = \mu_0(1 + \chi_m) \frac{I}{2\pi r} \hat{\phi}$$

Now we'll confirm this answer by finding  $\vec{M}$  and the associated bound currents and finally the  $\vec{B}$ -field generated  $\vec{J}_b$ ,  $\vec{K}_b$  and  $I_{\text{free}}$ .

$$\vec{M} = \chi_m \vec{H} = \frac{\chi_m I}{2\pi r} \hat{\phi}$$



On the inner surface

$$\vec{K}_b = \vec{M} \times \hat{n} = \frac{\chi_m I}{2\pi a} \hat{z}$$

$$\text{net}_{\text{bound}} \text{ current on inner surf} = 2\pi a |\vec{K}_b| = \chi_m I$$

the sum of bound current & free current is  $I + \chi_m I$

On the outer surface

$$\vec{K}_b = \vec{M} \times \hat{n} = -\frac{\chi_m I}{2\pi b} \hat{z}$$

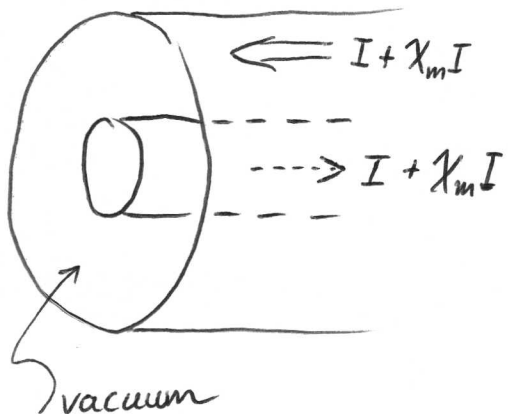
$$\text{net bound current on out surf} = 2\pi b |\vec{K}_b| = -\chi_m I$$

the sum of bound current & free current is  $-I - \chi_m I$

In the bulk of the linear material

$$\vec{J}_b = \nabla \times \vec{M} = \nabla \times \left( \frac{\chi_m I}{2\pi r} \hat{\phi} \right) = 0$$

Now we can draw an analogous system that will generate the same  $\vec{B}$ -field



$$B = \frac{\mu_0 (1 + \chi_m) I}{2\pi r} \hat{\phi}$$