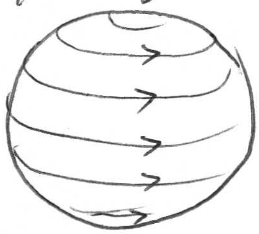
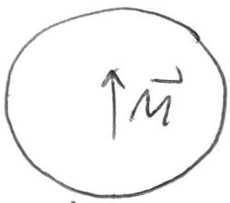
Homework #3 PH 632 2016

spinning sphere (hollow)



$\vec{K} = \sigma \vec{v} = \sigma \omega R \sin\theta \, \hat{\phi}$




$\vec{K}_b = M \sin\theta \, \hat{\phi}$

uniformly magnetized sphere (solid) with volume $V = \frac{4}{3}\pi R^3$

These objects produce the same $\vec{B}$-field if the magnetization ~~which~~ is $\vec{M} = \sigma \omega R \hat{z}$, (net dipole moment $\boxed{\vec{M} V = \frac{4}{3}\pi R^3 \sigma \omega R \hat{z}}$)

Expect the ~~m~~ $\vec{B}$-field outside ~~sphere~~ in the far-field limit to be

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{m}{r^3} \left( 2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta} \right) \quad \text{for } r > R$$

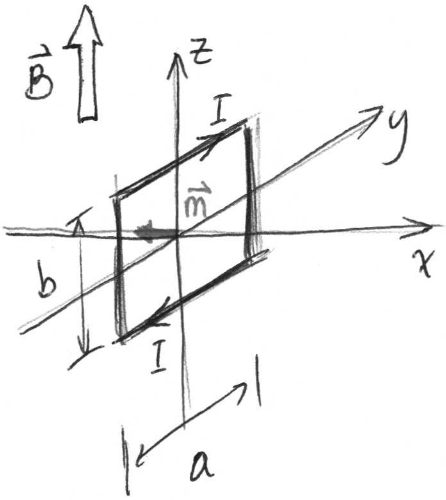
(spinning hollow sphere is a special case because the near field is also described by this eq$^n$)

$$\boxed{\vec{B}_{outside} = \frac{\mu_0}{\cancel{4\pi}} \frac{4}{3}\cancel{\pi} R^3 \sigma \omega R \frac{1}{r^3} \left( 2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta} \right)}$$

$\boxed{\vec{m} = \frac{Q}{2M} \vec{L}}$ is always true for classical ~~objects~~ charged objects.

In this case $Q = 4\pi R^2 \sigma$

$$\text{Expect } \boxed{\vec{L} = \frac{2M}{\cancel{4\pi R^2 \sigma}} \frac{4}{3}\pi R^3 \cancel{\sigma} \omega R \hat{z} = \frac{2}{3} M \omega R^2 \hat{z}}$$




Start with a simple case

$$\vec{m} = -Iab\,\hat{x}$$

$$\vec{B} = B_0\,\hat{z}$$

$$F_{top} = IaB\,\hat{x}$$

$$F_{bottom} = -IaB\,\hat{x}$$

Net torque $\quad \vec{\tau} = \left(IaB\frac{b}{2} + IaB\frac{b}{2}\right)\hat{y}$

$$= mB\,\hat{y}$$

More generally $\quad \vec{\tau} = \vec{m}\times\vec{B}$


$$W = -\int \vec{\tau}\cdot d\vec{\theta}$$

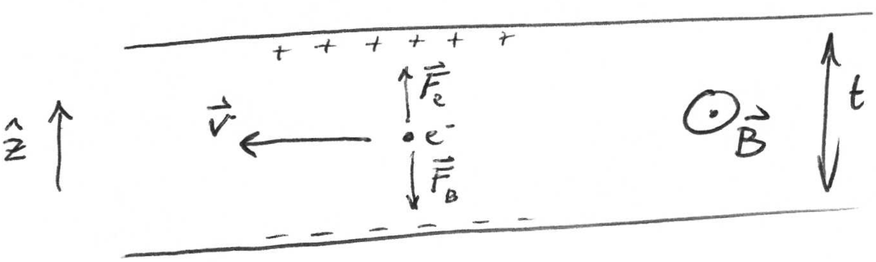
$$= -\int_{\pi/2}^{\theta_f} mB\sin\theta\, d\theta$$

$$= -\left[mB\cos\theta\right]_{\pi/2}^{\theta_f} = -mB\cos\theta_f = -\vec{m}\cdot\vec{B}$$

Energy stored in system (relative to $\theta = \pi/2$) is $-\vec{m}\cdot\vec{B}$

The conventional current is right moving

$\Rightarrow$ $e^-$ moves to left




$$\vec{F}_B = -e\vec{v}\times\vec{B} = -evB\,\hat{z}$$

$$\vec{F}_E = -\vec{F}_B = +evB\,\hat{z} = -e\vec{E}$$

Electric field.

$$\Rightarrow \vec{E} = -vB\,\hat{z}$$

$$|V_H| = \left|\int \vec{E}\cdot d\vec{l}\right| = vBt$$

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The current density in the material is easy to measure

$$J = \frac{I}{wt}$$

and we know $J = nev$

$$\Rightarrow \boxed{v = \frac{I}{wtne}}$$

To operate a Hall sensor for measuring B-field you need to know $\frac{1}{wtne}$, i.e. The proportionality const between $v$ & $I$.

Every material will have a different concentration of electrons, $n$.