Spinning sphere (hollow)

$$\vec{K} = \sigma \vec{v} = \sigma \omega R \sin\theta \, \hat{\phi}$$

$$\vec{K}_b = M \sin\theta \, \hat{\phi}$$

uniformly magnetized sphere (solid) with volume $V = \frac{4}{3}\pi R^3$

These objects produce the same $\vec{B}$-field if the magnetization ~~is~~ is $\vec{M} = \sigma\omega R \, \hat{z}$, (net dipole moment

$$\boxed{\vec{M} V = \frac{4}{3}\pi R^3 \sigma\omega R \, \hat{z}})$$

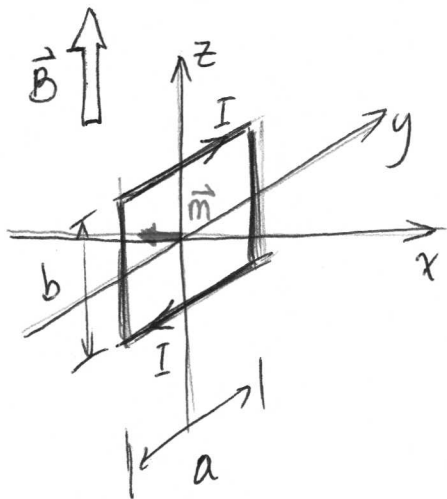Expect the ~~~~ $\vec{B}$-field outside ~~~~ in the far-field limit to be

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{m}{r^3}\left(2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta}\right) \qquad \text{for} \quad r > R$$

(spinning hollow sphere is a special case because the near field is also described by this eqⁿ)

$$\boxed{\vec{B}_{outside} = \frac{\mu_0}{4\pi} \frac{4}{3}\pi R^3 \sigma\omega R \, \frac{1}{r^3}\left(2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta}\right)}$$

$$\boxed{\vec{m} = \frac{Q}{2M}\vec{L}}$$ is always true for classical ~~objects~~ charged objects. In this case $Q = 4\pi R^2 \sigma$

Expect $$\boxed{\vec{L} = \frac{2M}{4\pi R^2 \sigma} \frac{4}{3}\pi R^3 \sigma\omega R \, \hat{z} = \frac{2}{3}M\omega R^2 \, \hat{z}}$$

Start with a simple case

$$\vec{m} = -Iab\,\hat{x}$$
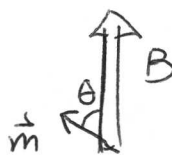
$$\vec{B} = B_0\,\hat{z}$$

$$F_{top} = IaB\,\hat{x}$$

$$F_{bottom} = -IaB\,\hat{x}$$

Net torque

$$\vec{\tau} = \left(IaB\frac{b}{2} + IaB\frac{b}{2}\right)\hat{y}$$

$$= mB\,\hat{y}$$

More generally $\quad \vec{\tau} = \vec{m} \times \vec{B}$

$$W = -\int \vec{\tau}\cdot d\vec{\theta}$$

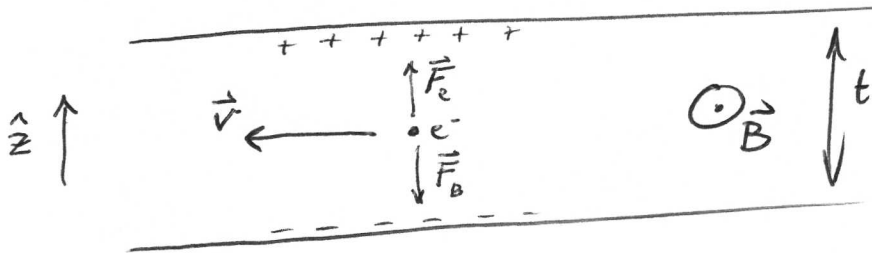$$= -\int_{\pi/2}^{\theta_f} mB\sin\theta\,d\theta$$

$$= -\left[mB\cos\theta\right]_{\pi/2}^{\theta_f} = -mB\cos\theta_f = -\vec{m}\cdot\vec{B}$$

Energy stored in system (relative to $\theta = \pi/2$) is $\quad -\vec{m}\cdot\vec{B}$

The conventional current is right moving

$\Rightarrow e^-$ moves to left

$$\vec{F_B} = -e\vec{v} \times \vec{B} = -evB\,\hat{z}$$

$$\vec{F_E} = -\vec{F_B} = +evB\,\hat{z} = -e\vec{E}$$

← Electric field.

$$\Rightarrow \vec{E} = -vB\,\hat{z}$$

$$\left| V_H \right| = \left| \int \vec{E} \cdot \vec{dl} \right| = vBt$$

The current density in the material is easy to measure

$$J = \frac{I}{wt}$$

and we know $\quad J = nev$

$$\Rightarrow \boxed{V = \frac{I}{wt\,ne}}$$

To operate a Hall sensor for measuring B-field you need to know $\frac{1}{wt\,ne}$, i.e. The proportionality const between v & I. Every material will have a different concentration of electrons, n.