PH632 – Winter 2016
Homework #3
Due Friday Jan 29 at 5pm

1. Magnetic dipole moment of a spinning object

In the first homework you found the magnetic vector potential outside a spinning hollow sphere of radius *R* and mass *M* that carries a uniform charge density σ:

$$\mathbf{A} = \frac{\mu_0 R^4 \omega\sigma}{3}\frac{\sin\theta}{r^2}\hat{\boldsymbol{\phi}}$$

a) Calculate the magnetic field outside the spinning hollow sphere?

b) Notice that the same magnetic field can be generated by a magnetic dipole, **m**, placed at the origin. Find **m**.

c) What is the angular momentum, **L**, of the spinning sphere?

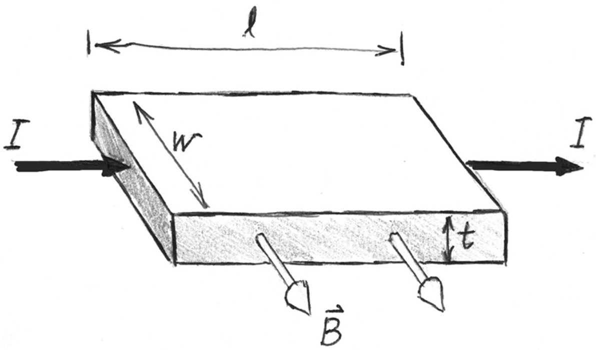
d) Write **m** in terms of **L**.

2. Consider a rectangular loop carrying current *I* in a uniform magnetic field **B**. The sides of the rectangular loop are length *a* and *b*. The long axis of the rectangle is perpendicular to **B**.

a) Show that the net torque on the loop is **m** x **B**, where **m** is the dipole moment of the loop.

b) If **m** and **B** are initially perpendicular, show that the work done to rotate the **m** to a different orientation is W = **m**·**B**. (Note, the potential energy of this new configuration will be -**m**·**B**).

3. To measure the magnetic field at the surface of a neodymium magnet we used a Hall effect sensor in Prof. Tate's lab. The following question explores the Hall effect (named after Edwin Hall).



A current density $J = nev$ (where n is the number density of charges, e is the elementary charge and v is the average velocity of charge) flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field **B** pointing out of the page. Charges moving through the bar are they deflected by **B**. This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which then produces an electric field to counteract the magnetic one.

a) Find the resulting potential difference, V_{Hall}, between the top and bottom of the bar, in terms of B, v, and the relevant dimensions of the bar.

b) A Hall effect sensor must know both V_{Hall} and v to determine magnetic field. How could a Hall effect sensor determine v?