

Wire 1 generates an electric field

$$E(s) = \frac{\lambda}{2\pi\epsilon_0 s}$$

Wire 1 generates a magnetic field

$$B(s) = \frac{\mu_0 I}{2\pi s} = \frac{\mu_0 v \lambda}{2\pi s}$$

Consider a segment of wire 2 of length l
the electric force on this segment is

$$F_E = QE = \lambda l \frac{\lambda}{2\pi\epsilon_0 d} = \frac{\lambda^2 l}{2\pi\epsilon_0 d}$$

The magnetic force on this segment is

$$F_B = QvB = \lambda l v \frac{\mu_0 v \lambda}{2\pi d} = \frac{\mu_0 \lambda^2 l}{2\pi d} v^2$$

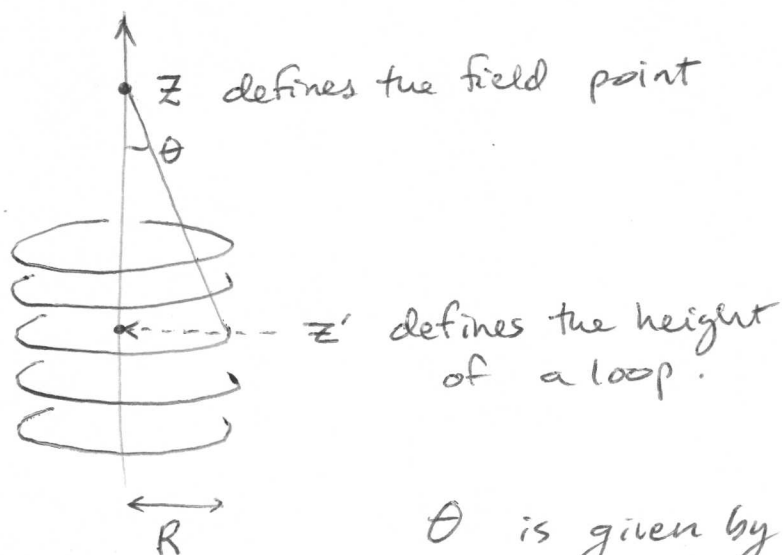
These forces balance when

$$F_E = F_B$$

$$\frac{1}{\epsilon_0} = \mu_0 v^2$$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$$

The wire's velocity will always be less than c , therefore, magnetic attraction will never fully balance electrostatic repulsion.



θ is given by $\tan \theta = \frac{R}{(z-z')}$

The B-field at z is given by

$$B(z) = \frac{\mu_0 I}{2} \int_{z' = z_{\text{bottom}}}^{z_{\text{top}}} \frac{R^2}{(R^2 + (z-z')^2)^{3/2}} n dz'$$

$$= \frac{\mu_0 I}{2} \int \frac{R^2}{R^3 \left(1 + \frac{(z-z')^2}{R^2}\right)^{3/2}} n dz'$$

$$= \frac{\mu_0 I n}{2R} \int \frac{1}{\left(1 + \frac{1}{\tan^2 \theta}\right)^{3/2}} dz'$$

To complete the integral I need to write dz' in terms of θ and $d\theta$.

$$z' = z - \frac{R}{\tan \theta}$$

$$\frac{dz'}{d\theta} = -R \left(1 + \frac{1}{\tan^2 \theta}\right)$$

$$\Rightarrow B(z) = \frac{\mu_0 I n}{2R} \int_{\theta_{\text{bottom}}}^{\theta_{\text{top}}} \frac{1}{\left(1 + \frac{1}{\tan^2 \theta}\right)^{3/2}} - R \left(1 + \frac{1}{\tan^2 \theta}\right) d\theta$$

$$= -\frac{\mu_0 I n}{2} \int \frac{1}{\left(1 + \frac{1}{\tan^2 \theta}\right)^{1/2}} d\theta$$

Notice that $1 + \frac{1}{\tan^2 \theta} = 1 + \frac{\cos^2 \theta}{\sin^2 \theta}$

$$= \frac{\cancel{\sin^2 \theta}}{\sin^2 \theta} (\sin^2 \theta + \cos^2 \theta)$$

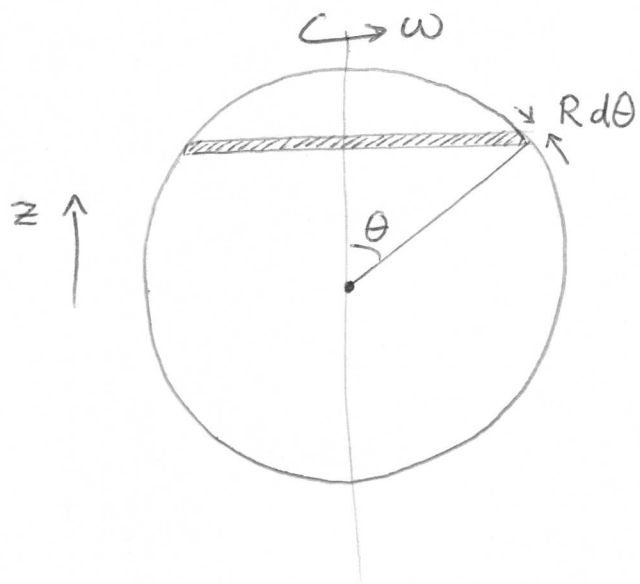
$$= \frac{1}{\sin^2 \theta}$$

$$\Rightarrow B(z) = -\frac{\mu_0 I n}{2} \int \frac{1}{(\sin^2 \theta)^{1/2}} d\theta$$

$$= -\frac{\mu_0 I n}{2} \int \sin \theta d\theta$$

$$= \frac{\mu_0 I n}{2} \left[\cos \theta \right]_{\theta_{\text{bottom}}}^{\theta_{\text{top}}}$$

$$= \frac{\mu_0 I n}{2} (\cos \theta_{\text{top}} - \cos \theta_{\text{bottom}})$$



The linear charge density on the shaded ring is $\sigma R d\theta$

The effective current on the shaded ring is $I = \lambda v$

$$= \sigma R d\theta \omega R \sin\theta$$

$$= \sigma \omega R^2 \sin\theta d\theta$$

At the center of the hollow sphere, the shaded ring creates a B-field (pointing in z direction)

$$\frac{\mu_0 I}{2} \frac{(R \sin\theta)^2}{\left((R \sin\theta)^2 + (R \cos\theta)^2 \right)^{3/2}} = \frac{\mu_0 I}{2R} \sin^2\theta$$

Now, integrating over the whole sphere

$$B_{\text{center}} = \frac{\mu_0}{2R} \int_{\theta=0}^{\pi} \sin^2\theta \cancel{I} \sigma \omega R^2 \sin\theta d\theta$$

$$= \frac{\mu_0 \sigma \omega R}{2} \int_0^{\pi} \sin^3\theta d\theta$$

$= \frac{4}{3}$

$$= \frac{2}{3} \mu_0 \sigma \omega R \quad \text{in } z\text{-direction}$$