PH632 HW #2

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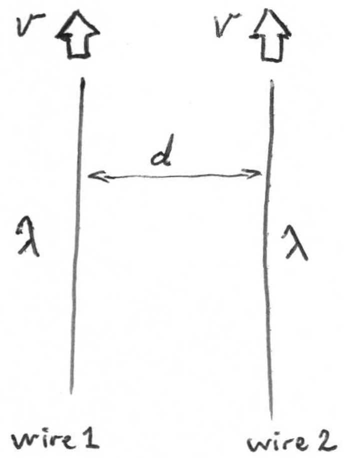


Wire 1 generates an electric field

$$E(s) = \frac{\lambda}{2\pi\epsilon_0 s}$$

Wire 1 generates a magnetic field

$$B(s) = \frac{\mu_0 I}{2\pi s} = \frac{\mu_0 v \lambda}{2\pi s}$$

Consider a segment of wire 2 of length l

The electric force on this segment is

$$F_E = QE = \lambda l \frac{\lambda}{2\pi\epsilon_0 d} = \frac{\lambda^2 l}{2\pi\epsilon_0 d}$$

The magnetic force on this segment is

$$F_B = QvB = \lambda l v \frac{\mu_0 v \lambda}{2\pi d} = \frac{\mu_0 \lambda^2 l}{2\pi d} v^2$$

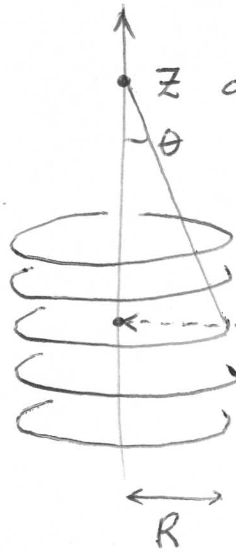
These forces balance when

$$F_E = F_B$$

$$\frac{1}{\epsilon_0} = \mu_0 v^2$$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$$

The wire's velocity will always be less than c, therefore, magnetic attraction will never fully balance electrostatic repulsion.



z defines the field point

z' defines the height of a loop.

θ is given by $\quad \tan\theta = \dfrac{R}{(z-z')}$

The B-field at z is given by

$$B(z) = \frac{\mu_0 I}{2} \int_{z'=z_{bottom}}^{z_{top}} \frac{R^2}{\left(R^2 + (z-z')^2\right)^{3/2}} \, n\,dz'$$

$$= \frac{\mu_0 I}{2} \int \frac{R^2}{R^3\left(1 + \frac{(z-z')^2}{R^2}\right)^{3/2}} \, n\,dz'$$

$$= \frac{\mu_0 I n}{2R} \int \frac{1}{\left(1 + \cancel{\tan}\,\frac{1}{\tan^2\theta}\right)^{3/2}} \, dz'$$

To complete the integral I need to write dz' in terms of θ and $d\theta$.

$$z' = z - \frac{R}{\tan\theta}$$

$$\frac{dz'}{d\theta} = -R\left(1 + \frac{1}{\tan^2\theta}\right)$$

$$\Rightarrow \quad B(z) = \frac{\mu_0 I n}{2R} \int_{\theta_{bottom}}^{\theta_{top}} \frac{1}{\left(1 + \frac{1}{\tan^2\theta}\right)^{3/2}} - R\left(1 + \frac{1}{\tan^2\theta}\right) d\theta$$

$$= -\frac{\mu_0 I n}{2} \int \frac{1}{\left(1 + \frac{1}{\tan^2\theta}\right)^{1/2}} d\theta$$

Notice that $1 + \frac{1}{\tan^2\theta} = 1 + \frac{\cos^2\theta}{\sin^2\theta}$

$$= \frac{1}{\sin^2\theta}\left(\sin^2\theta + \cos^2\theta\right)$$

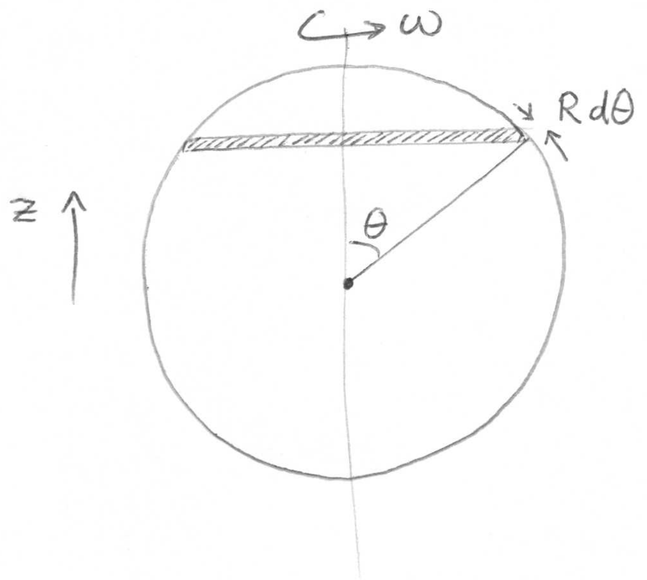
$$= \frac{1}{\sin^2\theta}$$

$$\Rightarrow \quad B(z) = -\frac{\mu_0 I n}{2} \int \frac{1}{\left(\sin^2\theta\right)^{-1/2}} d\theta$$

$$= -\frac{\mu_0 I n}{2} \int \sin\theta \, d\theta$$

$$= \frac{\mu_0 I n}{2} \Big[\cos\theta \Big]_{\theta_{bottom}}^{\theta_{top}}$$

$$= \frac{\mu_0 I n}{2} \left(\cos\theta_{top} - \cos\theta_{bottom} \right)$$



The linear charge density on the shaded ring is $\sigma R\,d\theta$

The effective current on the shaded ring is

$$I = \lambda v$$
$$= \sigma R\,d\theta\,\omega R\sin\theta$$
$$= \sigma\omega R^2\sin\theta\,d\theta$$

At the center of the hollow sphere, the shaded ring creates a B-field (pointing in z direction)

$$\frac{\mu_0 I}{2}\frac{(R\sin\theta)^2}{\left((R\sin\theta)^2+(R\cos\theta)^2\right)^{3/2}} = \frac{\mu_0 I}{2R}\sin^2\theta$$

Now, integrating over the whole sphere

$$B_{center} = \frac{\mu_0}{2R}\int_{\theta=0}^{\pi}\sin^2\theta\,\cancel{I(\theta)}\,\sigma\omega R^2\sin\theta\,d\theta$$

$$= \frac{\mu_0\sigma\omega R}{2\cancel{R}}\underbrace{\int_0^{\pi}\sin^3\theta\,d\theta}_{=4/3}$$

$$= \frac{2}{3}\mu_0\sigma\omega R \quad \text{in } z\text{-direction}$$