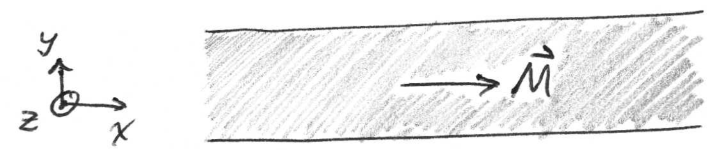
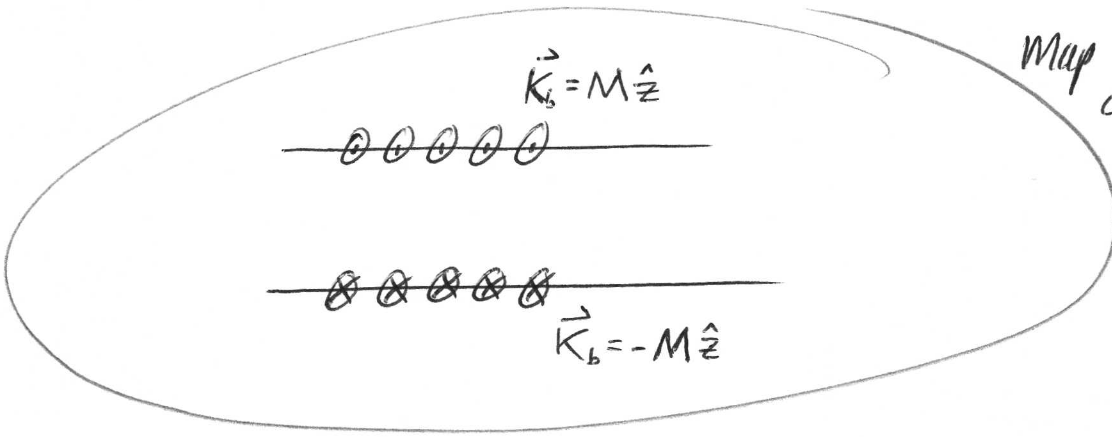
PH 632

DAY 9

Instructor
Ethan Minot

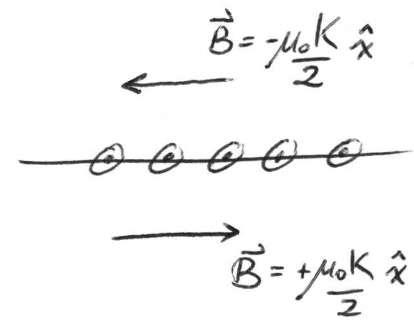
Review the pop quiz from last time



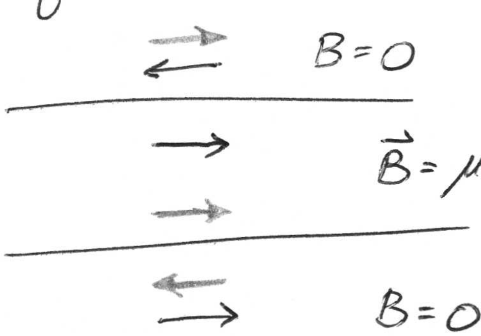


Map onto a current distribution that produces the same $\vec{B}$-field.

B-field around an infinite sheet of current



Superposition of $\vec{B}$-fields from the 2 sheets of current



$$\vec{B} = \mu_0 K\hat{x} = \mu_0 M\hat{x} = \mu_0\vec{M}$$

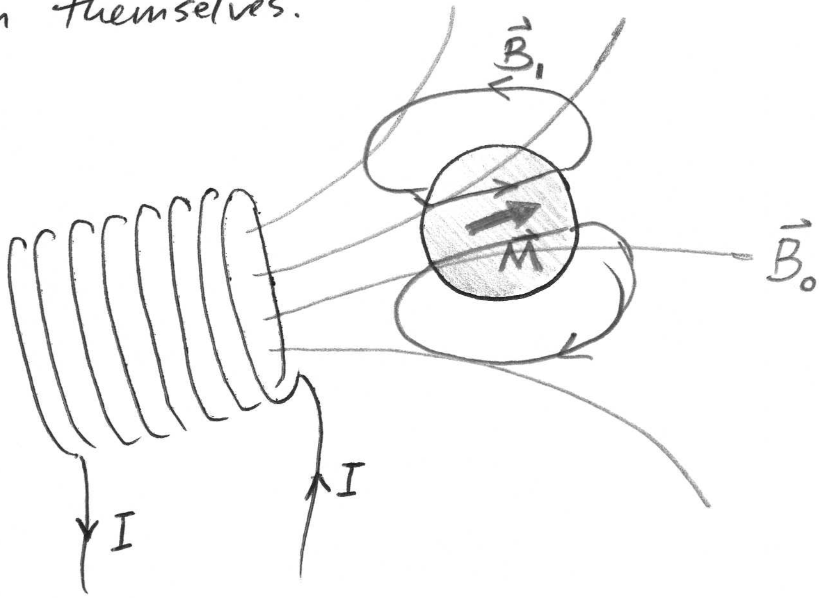
FREE CURRENTS MAGNETIZE LINEAR MATERIALS

$$\begin{tikzcd} \vec{J}_{free} \arrow[r, bend left, "\text{generates}"] & \vec{B} \arrow[r, bend left, "\text{magnetizes material}"] & \vec{M} \arrow[l, bend left, "\text{generates}"] \end{tikzcd}$$

In this section of the course we'll be discussing paramagnetic and diamagnetic materials (<u>not</u> ferromagnetic).

Paramagnetic & diamagnetic materials only become magnetized in the presence of an external $\vec{B}$ field.

However, once they are magnetized, the have a back action on themselves.



$\vec{B}_0$ generated by the current I

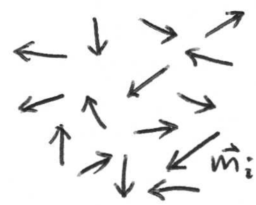
$\vec{B}_1$ generated by the magnetized material.

$\vec{B}_0$ & $\vec{B}_1$ add together inside the object.
We need to find a self-consistent solution for $\vec{M}$.

The problem is analytically tractable if $\vec{M} \propto \vec{B}$

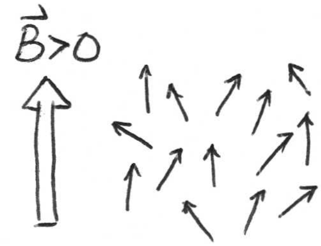
Linear Paramagnetic Material

$$\vec{M} = \begin{pmatrix}\text{positive} \\ \text{constant}\end{pmatrix} \vec{B}$$



random orientation

$$\vec{M} = \frac{\sum \vec{m}_i}{\Delta V} = 0$$



partial alignment

$$|\vec{M}| > 0$$

Linear Diamagnetic Material

$$\vec{M} = \begin{pmatrix}\text{negative} \\ \text{constant}\end{pmatrix} \vec{B}$$

Save discussion of physical mechanisms until later. Interesting quantum mechanics.

How do we find ~~a~~ the self-consistent value for $\vec{M}$ without doing an ~~m~~ iterative numerical approach?

__Answer:__ Solve for the combined quantity $\frac{\vec{B}}{\mu_0} - \vec{M}$ first.

Then deduce $\vec{M}$.

(Just like in electrostatics when we solved for $\epsilon_0 \vec{E} + \vec{P}$)

What is special about $\frac{\vec{B}}{\mu_0} - \vec{M}$?

Answer: With no linear material present, magneto statics is governed by

$$\nabla \times \vec{B} = \mu_0 \vec{J}_{free} \qquad \text{——— } (1)$$

when we add linear materials, $\nabla \times \vec{M}$ behaves like an additional current density

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_{free} + \nabla \times \vec{M} \right)$$

Re arrange this eq^n

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_{free} \qquad \text{——— } (2)$$

All the techniques we used to solve (1)
(Biot-Savart Law, Ampere's Law etc.)
can be reformulated for eq^n (2).