PH632

DAY 8

Instructor:
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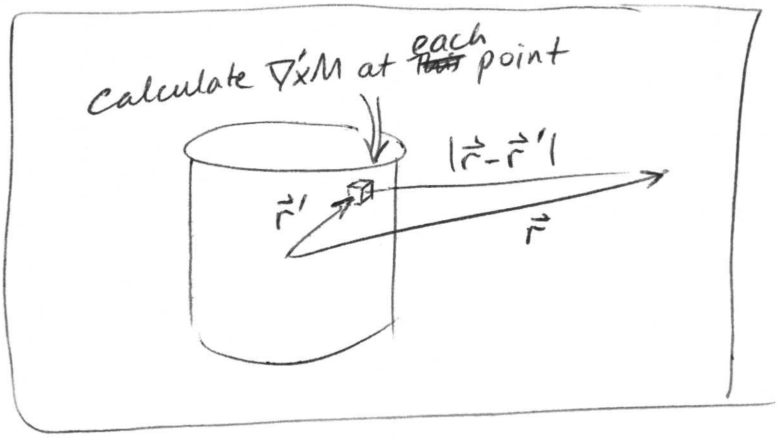
Summary from last time:

The net vector potential from summing up all the dipole fields in a magnetized material is

$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\substack{\text{all} \\ \text{space}}} \frac{\nabla' \times \vec{M}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3\vec{r}\,'$$



Compare to the vector potential generated by a current density $\vec{J}(\vec{r}\,')$

$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\substack{\text{all} \\ \text{space}}} \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3\vec{r}\,'$$

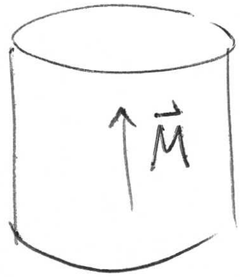
Beautiful result:

A current density $\vec{J} = \nabla \times \vec{M}$ will generate the same field as the magnetization $\vec{M}$.

(~~Also~~ very useful result, because we already have tools to find $\vec{B}$ given an arbitrary $\vec{J}$).

APPLICATION EXAMPLE

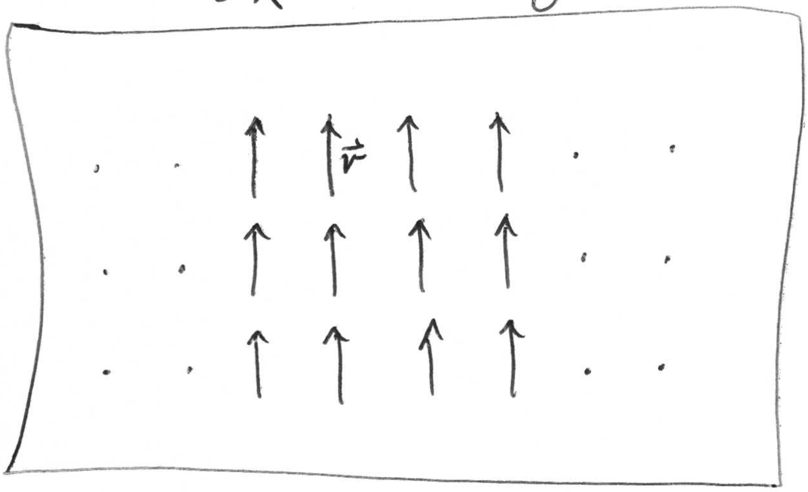
Estimate $\vec{B}$ ~~outside~~ on the surface of a neodymium permanent magnet



$$\vec{M} = \begin{cases} 3\times10^{5}\,\frac{J}{Tm^{3}}\,\hat{z} & \text{inside} \\ 0 & \text{outside} \end{cases}$$

$$\nabla\times\vec{M} = \begin{cases} 0 & \text{inside} \quad \left(\text{because } \vec{M} \text{ is constant inside}\right) \\ \text{Diverges} & \text{on surface} \\ 0 & \text{outside} \end{cases}$$

To visualize $\nabla\times\vec{M}$, consider an airflow with velocity $\vec{v}$ described by the same vector field pattern as $\vec{M}$

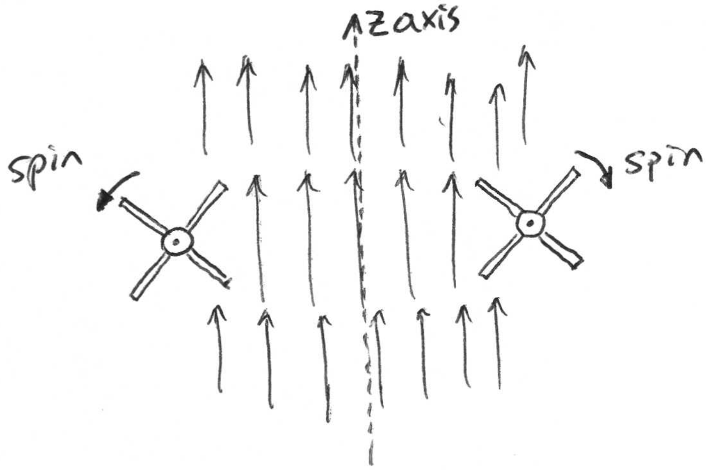


Place a very small toy paddle wheel in this airflow.

If the paddle wheel spins, you've found a position ~~with~~ where $\nabla\times\vec{v}$ is non-zero.

When the axle of the paddle wheel points in the direction of $\nabla\times\vec{v}$, the wheel spins the fastest.

Demonstration with leaf blower and paddle wheel.



Paddle wheel spins when it is on the edge of the flow.

Curl points in $\hat{\phi}$ direction.

If $\vec{M}$ abruptly drops to zero at a surface, the direction of $\nabla \times \vec{M}$ is $\vec{M} \times \hat{n}$, where $\hat{n}$ is the unit vector normal to the surface.

To evaluate $\vec{A}(\vec{r})$, we break up the integral into 3 regions

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\substack{\text{all} \\ \text{space}}} \frac{\nabla \times M}{|\vec{r}-\vec{r}\,'|} \, d^3\vec{r}\,'$$

$$= \frac{\mu_0}{4\pi} \int_{\substack{\text{inside} \\ \text{object}}} \frac{\nabla \times M}{|\vec{r}-\vec{r}\,'|} \, d^3\vec{r}\,' + \frac{\mu_0}{4\pi} \int_{\substack{\text{surface} \\ \text{of object}}} \frac{\vec{M} \cdot \hat{n}}{|\vec{r}-\vec{r}\,'|} \, da' + \cancelto{0}{\frac{\mu_0}{4\pi} \int_{\substack{\text{outside} \\ \text{object}}} \frac{\nabla \times M}{|\vec{r}-\vec{r}\,'|} \, d^3\vec{r}\,'}$$

$\nabla \times \vec{M}$ is called bound current density, $\vec{J}_b$.
$\vec{M} \cdot \hat{n}$ is called bound sheet current, $\vec{K}_b$.