PH 632

DAY 7

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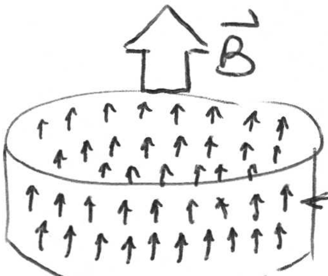

millions of individual dipoles, $\vec{m}$.

Consider a neodymium permanent magnet, each unpaired electron has $|\vec{m}| \approx \frac{e\hbar}{2m_e} \approx 10^{-23}\, J/T$

How do I calculate $\vec{B}$ at the surface?

First thought:

Superimpose the $\vec{B}$-fields of each dipole

One dipole at origin pointing $\hat{z}$	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{m}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$ — ①
Pointing in arbitrary direction	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left(3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right)$ — ②
at point $\vec{r}\,'$	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{\lvert\vec{r}-\vec{r}\,'\rvert^3}\left(\frac{3\,\vec{m}\cdot(\vec{r}-\vec{r}\,')(\vec{r}-\vec{r}\,')}{\lvert\vec{r}-\vec{r}\,'\rvert^2} - \vec{m}\right)$

Exercise: Show that ② reduces to ① when $\vec{m} = m\hat{z}$

The superposition of many dipole fields is given by

$$\vec{B}(\vec{r}) = \sum_{\vec{r}_i'} \frac{\mu_0}{4\pi} \frac{1}{|\vec{r}-\vec{r}_i'|^3} \left(\quad \cdots \quad \right)$$

location of ith dipole

We turn the sum into an integral by grouping dipoles together into small pixels of space, $d^3\vec{r}'$.

Within one pixel there are $n(\vec{r}')\,d^3\vec{r}'$ dipoles.

The concentration of dipoles.

$$\vec{B}(\vec{r}) = \int_{\substack{\text{all}\\ \text{space}}} \frac{\mu_0}{4\pi} \frac{1}{|\vec{r}-\vec{r}'|^3} \left[\frac{3\, n(\vec{r}')\vec{m}\cdot(\vec{r}-\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2} - n(\vec{r}')\vec{m} \right] d^3\vec{r}'$$

The quantity $n(\vec{r}')\vec{m} = \vec{M}$ is called the magnetization of the material.

Analogous to electric polarization of a material.

Pop Quiz – Day 7

Estimate **M** inside a neodymium magnet. (The interatomic spacing is about 0.3 nm).

The integral describing the superposition of magnetic dipole B-fields is a bear. We don't have analytic techniques to move forward. What to do?

Last quarter we learned how to deal with superposition of electric dipole fields

$\boxed{\text{Step 1}}$ Potential generated by 1 electric dipole

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$\boxed{\text{Step 2}}$ Super position of millions of dipole field potentials

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{P}(\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, d^3\vec{r}\,'$$

$$= \frac{1}{4\pi\epsilon_0} \int \vec{P}(\vec{r}\,')\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) d^3\vec{r}\,'$$

$\boxed{\text{Step 3}}$ Use integration by parts to solve the integral.

We will try this approach with magnetic dipoles.