

## MAGNETIC DIPOLE MOMENTS (continued)

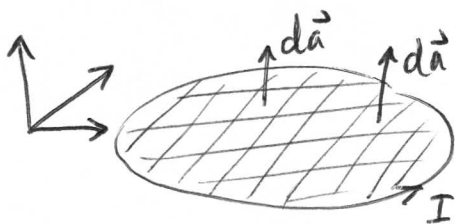
General expression for  $\vec{m}$  generated by an arbitrary shaped loop of  $I$  current (derived in Griffiths)

$$\vec{m} = I \int d\vec{a}$$

The area elements  $d\vec{a}$  cover the surface that is bounded by the current loop.

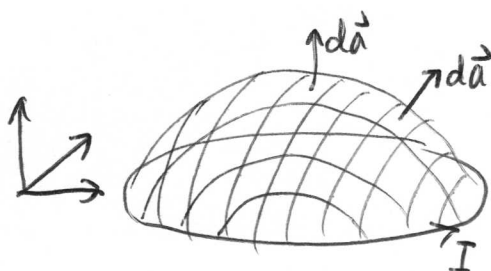
There is often more than one way to choose the surface. All surfaces will integrate to the same value.

For example



choice 1

$$\int_1 d\vec{a} = \pi R^2$$

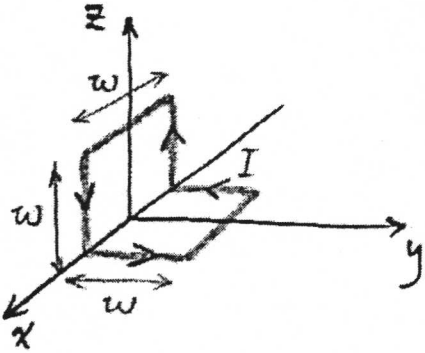


choice 2

$$\int_2 d\vec{a} = \pi R^2$$

## Pop Quiz - Day 6

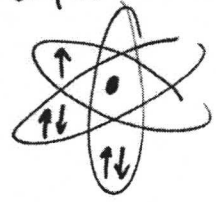
Find the magnetic dipole moment (direction and magnitude) of this current loop:



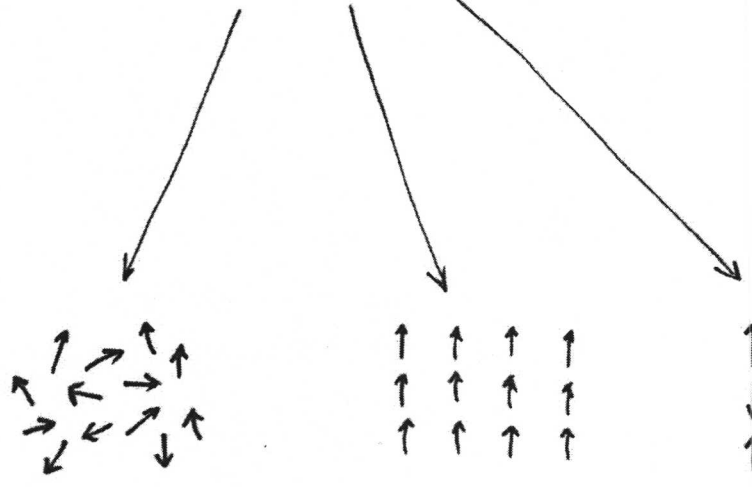
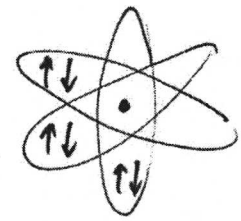
(If you have time,  
try ~~the~~ defining a  
different surface  
that is bounded  
by the current loop)

# Electrons generate magnetic dipole fields. This leads to the magnetic properties of materials

Materials with one or more unpaired electrons per atomic/molecular unit



Materials with fully paired electrons per atomic/molecular unit.



- Paramagnetic (dipoles will align parallel to an external B field)
- Example  $O_2$  gas or liquid.

- Ferromagnetic (like iron, dipoles are correlated, even without  $B_{ext}$ )
- Example: Fe

- Anti Ferromagnetic (strong correlation between dipoles, but makes neighbors point in opposite directions)
- Example:  $Fe_2O_3$

- Diamagnetic
- Example: Water
- $B_{external}$  will induce dipoles anti-parallel to  $B_{ext}$ , (very weak effect)

Ferromagnetism & anti ferromag are both driven by a QM effect called exchange interaction.

④

# ESTIMATE THE B-FIELD PRODUCED BY A FERROMAGNET

How many dipoles per unit volume?

$$n = \frac{1}{(\text{Volume per atom})} \cong \frac{1}{(0.3\text{nm})^3} = 3 \times 10^{28} \text{ m}^{-3}$$

Strength of each dipole?

Each  $e^-$  has angular momentum  $L = \frac{\hbar}{2}$ .

There is a useful relationship between  $\vec{L}$  &  $\vec{m}$  for a uniformly charged classical, non-relativistic object:

$$\vec{m} = \frac{q}{2m} \vec{L}$$

classical result.

This result is independent of the object's geometry.

For electrons, the classical result breaks down.

Electrons are too small (spinning too fast).

$$|\vec{m}| = \underbrace{2.0023193}_{\text{prefactor}} \frac{e \hbar}{2m_e} \approx 10^{-23} \text{ J/T}$$

calculated using relativistic QM. Precision calculations by Feynman and others.