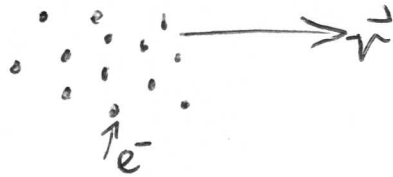


RELATIONSHIP BETWEEN CURRENT DENSITY & PARTICLE FLUX



A current density means that a cloud of electrons is moving through space.

$$(\text{Current density}) = (e) \cdot (\text{Particle Flux})$$

Particle flux ~~m~~ can be explained as follows

1d Imagine a line of conga dancers ~~m~~, 2 dancers per meter, moving at 1 m s^{-1} .

The "particle density" is $n_{1d} = 2 \text{ m}^{-1}$

The velocity is $v = 1 \text{ m s}^{-1}$

The ~~particle~~ 1d particle flux is ~~WAT~~ $n_{1d} v = 2 \text{ s}^{-1}$

Electrical equivalent: $I = \lambda v$
 \uparrow line charge density

(2)

2d | Imagine a pack of runners starting a marathon.

The "particle density" is $n_{2d} = 1 \text{ m}^{-2}$

The velocity is 2.5 ms^{-1}

The 2d particle flux is $n_{2d} v = 2.5 \text{ s}^{-1} \text{ m}^{-1}$

i.e. If I drew a ^{1-m-long} chalk line on the road perpendicular to \vec{v} ,

2.5 runners would step over that line every second.

Electrical equivalent: $\vec{K} = \sigma \vec{v}$

3d | Imagine a flock of birds in the sky

The "particle density" is $n_{3d} = 1 \text{ m}^{-3}$

The velocity is 10 ms^{-1}

The 3d particle flux is $n_{3d} v = 10 \text{ s}^{-1} \text{ m}^{-2}$

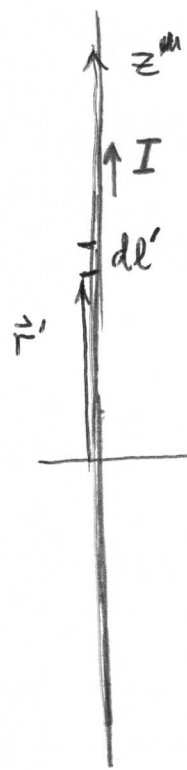
i.e. If I drew a 1-m^2 window in the sky, perpendicular to \vec{v} , 10 birds would fly through the window every second.

Electrical equivalent: $\vec{J} = \rho \vec{v}$

(3)

Application of the Biot-Savart law

Infinitely long straight wire



$$B(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \underset{\text{along wire}}{d\vec{l}'} \times \left(\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \right)$$

$$d\vec{l}' = dz' \hat{z}$$

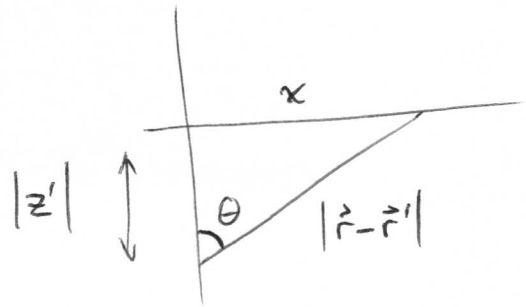
Integrating along wire means
 $z' = -\infty$ to $+\infty$.

Since I placed \vec{r} on the x-axis, I have $|\vec{r} - \vec{r}'| = \sqrt{z'^2 + x^2}$

\swarrow z-coordinate of source
 \nwarrow x-coord of field point.

We need an expression for the cross-product

$$\hat{z} \times (\vec{r} - \vec{r}')$$



$$\begin{aligned} \hat{z} \times (\vec{r} - \vec{r}') &= \sin \theta |\vec{r} - \vec{r}'| \hat{y} \\ &= \frac{x}{|\vec{r} - \vec{r}'|} |\vec{r} - \vec{r}'| \hat{y} = x \hat{y} \end{aligned}$$

④

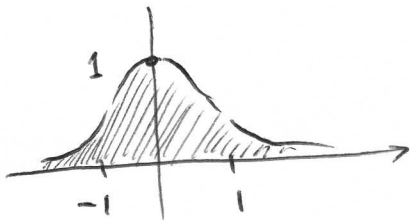
$$\Rightarrow \vec{B}(\vec{r}) = \frac{\mu_0 I x}{4\pi} \hat{y} \int_{-\infty}^{\infty} \frac{1}{(x^2 + z'^2)^{3/2}} dz'$$

To evaluate the integral, I want to make it dimensionless. The length x sets the scale. I will use $\frac{z'}{x}$ as the dimensionless variable.

$$\text{Integral} = \int_{-\infty}^{\infty} \frac{1}{x^3 \left(1 + \left(\frac{z'}{x} \right)^2 \right)^{3/2}} dz'$$

$$= \frac{1}{x^2} \int_{-\infty}^{\infty} \frac{1}{(1 + u^2)^{3/2}} du \quad \text{where } u = \frac{z'}{x} \\ dz' = x du$$

Sketch the integrand function



estimate area under curve.
~2.

"Like putting roller skates on a scary giant spider"

Final answer

$$\vec{B}(0,0,x) = \frac{\mu_0 I}{2\pi x} \hat{y}$$

which can be generalized by symmetry argument

$$\vec{B}(s) = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$