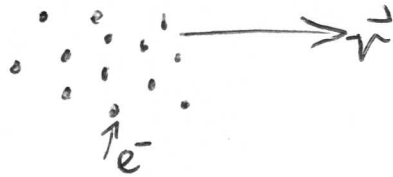


## RELATIONSHIP BETWEEN CURRENT DENSITY &amp; PARTICLE FLUX



A current density means that a cloud of electrons is moving through space.

$$(\text{Current density}) = (e) \cdot (\text{Particle Flux})$$

Particle flux  $n$  can be explained as follows

1d Imagine a line of conga dancers ~~mm~~, 2 dancers per meter, moving at  $1 \text{ ms}^{-1}$ .

The "particle density" is  $n_{1d} = 2 \text{ m}^{-1}$

The velocity is  $v = 1 \text{ ms}^{-1}$

The ~~particle~~ 1d particle flux is ~~mm~~  $n_{1d} v = 2 \text{ s}^{-1}$

Electrical equivalent:  $I = \lambda v$   
 $\uparrow$  line charge density

(2)

2d

Imagine a pack of runners starting a marathon.

The "particle density" is  $n_{2d} = 1 \text{ m}^{-2}$

The velocity is  $2.5 \text{ ms}^{-1}$

The 2d particle flux is  $n_{2d} v = 2.5 \text{ s}^{-1} \text{ m}^{-1}$

i.e. If I drew a <sup>1-m-long</sup> chalk line on the road perpendicular to  $\vec{v}$ ,  
2.5 runners would step over that line every second.

Electrical equivalent:  $\vec{K} = \sigma \vec{v}$

3d

Imagine a flock of birds in the sky

The "particle density" is  $n_{3d} = 1 \text{ m}^{-3}$

The velocity is  $10 \text{ ms}^{-1}$

The 3d particle flux is  $n_{3d} v = 10 \text{ s}^{-1} \text{ m}^{-2}$

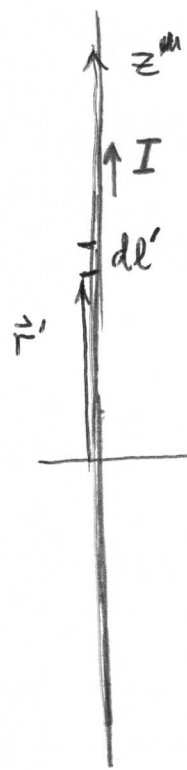
i.e. If I drew a  $1\text{-m}^2$  window in the sky, perpendicular to  $\vec{v}$ , 10 birds would fly through the window every second.

Electrical equivalent:  $\vec{J} = \rho \vec{v}$

(3)

# Application of the Biot-Savart law

Infinitely long straight wire



$$B(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \underset{\text{along wire}}{d\vec{l}'} \times \left( \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \right)$$

$$d\vec{l}' = dz' \hat{z}$$

$B(\vec{r}) = ?$

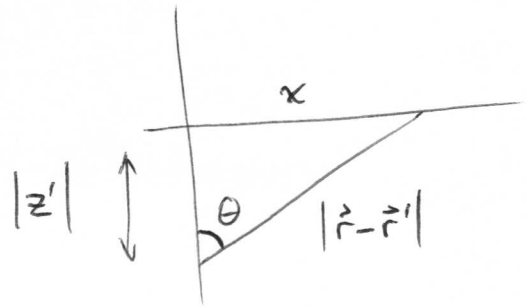
Integrating along wire means  $z' = -\infty$  to  $+\infty$ .

Since I placed  $\vec{r}$  on the x-axis, I have  $|\vec{r} - \vec{r}'| = \sqrt{z'^2 + x^2}$

$\swarrow$  z-coordinate of source  
 $\nwarrow$  x-coord of field point.

We need an expression for the cross-product

$$\hat{z} \times (\vec{r} - \vec{r}')$$



$$\begin{aligned} \hat{z} \times (\vec{r} - \vec{r}') &= \sin \theta |\vec{r} - \vec{r}'| \hat{y} \\ &= \frac{x}{|\vec{r} - \vec{r}'|} |\vec{r} - \vec{r}'| \hat{y} = x \hat{y} \end{aligned}$$

④

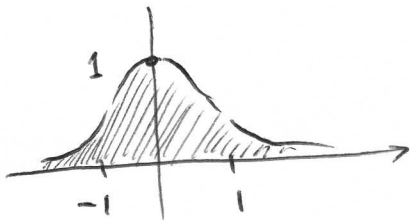
$$\Rightarrow \vec{B}(\vec{r}) = \frac{\mu_0 I x}{4\pi} \hat{y} \int_{-\infty}^{\infty} \frac{1}{(x^2 + z'^2)^{3/2}} dz'$$

To evaluate the integral, I want to make it dimensionless. The length  $x$  sets the scale. I will use  $\frac{z'}{x}$  as the dimensionless variable.

$$\text{Integral} = \int_{-\infty}^{\infty} \frac{1}{x^3 \left( 1 + \left( \frac{z'}{x} \right)^2 \right)^{3/2}} dz'$$

$$= \frac{1}{x^2} \int_{-\infty}^{\infty} \frac{1}{(1 + u^2)^{3/2}} du \quad \text{where } u = \frac{z'}{x} \\ dz' = x du$$

Sketch the integrand function



estimate area under curve.  
~2.

"Like putting roller skates on a scary giant spider"

Final answer

$$\vec{B}(0,0,x) = \frac{\mu_0 I}{2\pi x} \hat{y}$$

which can be generalized by symmetry argument

$$\vec{B}(s) = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$