PH632

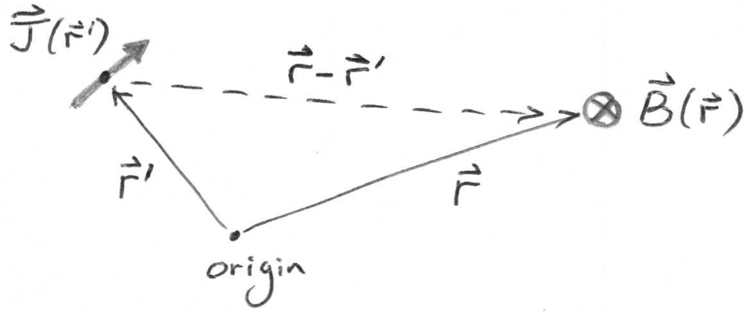
DAY 2

Instructor: Ethan Minot

Last time $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\text{all space}} \vec{J}(\vec{r}\,') \times \left(\frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3} \right) d^3\vec{r}\,'$

[Biot-Savart law for 3-d current density]

Diagram summarizing the relevant vectors

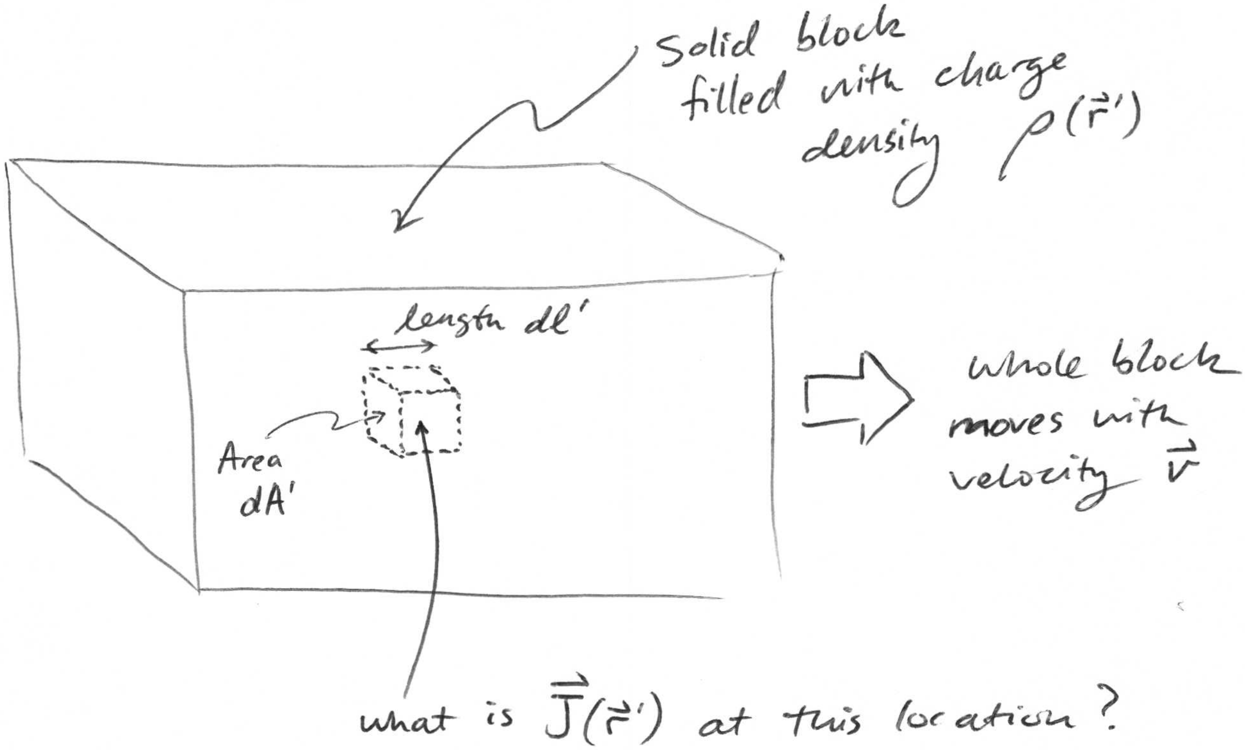


The cross-product $\vec{J} \times (\vec{r} - \vec{r}\,')$ determines the direction of $\vec{B}$ generated by $\vec{J}(\vec{r}\,')$.

~~Mo~~ CURRENT DENSITY CREATED BY ~~MOVING~~ MOTION OF A CHARGE DISTRIBUTION.

Sometimes you have to calculate $\vec{J}$ based on knowledge of charge density and velocity.

Example



A charge of $\rho(\vec{r}')\,dA'\,dl'$ passes through the window dA' in a time $\dfrac{\text{distance}}{\text{velocity}} = \dfrac{dl'}{v}$.

$$\Rightarrow \quad \underbrace{J\,dA'}_{\text{another way to call the current passing through the window.}} = \frac{\rho(\vec{r}')\,dA'\,dl'}{dl'/v} = \rho(\vec{r}')\,v\,dA'$$

$$\Rightarrow \quad \boxed{\vec{J} = \rho(\vec{r}')\,\vec{v}}$$

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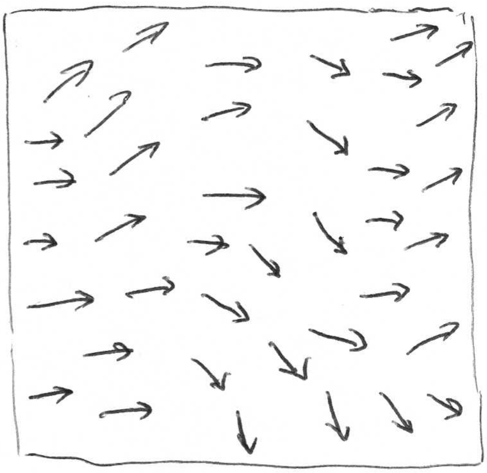
POP QUIZ

A line charge (linear charge density λ) is bent into a circle of radius R.

The ring of charge lies in the x-y plane, centered at the origin, and spins about the z-axis with angular frequency ω.

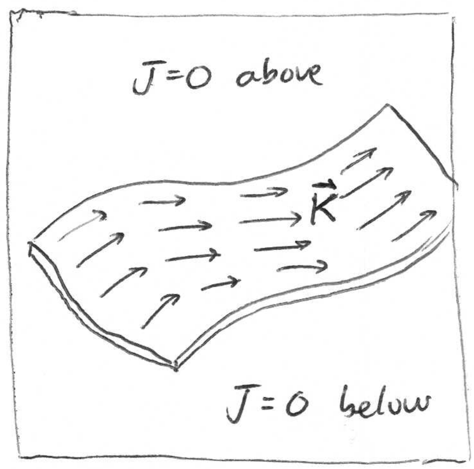
Find the magnitude of the current flowing in this system.

Sometimes current flow is restricted to thin sheets or thin wires.



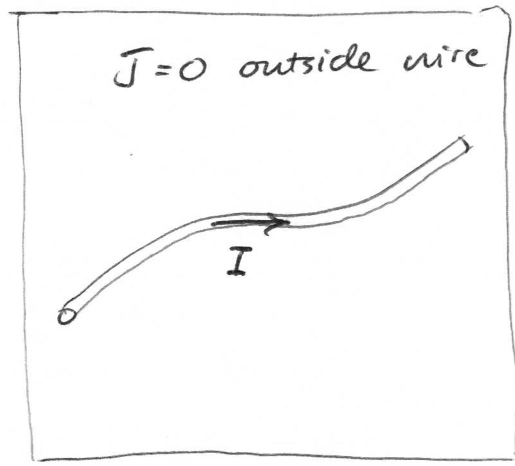
$\vec{J} > 0$ everywhere

For example, the ionized gas swirling around inside a star.



Current restricted to a thin sheet.

For example, electrons moving through a metallic film.



Current restricted to thin wire.

3d integral

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\text{all space}} \vec{J}(\vec{r}\,') \times \left(\qquad \right) d^3\vec{r}\,'$$

2d interal

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\text{surface}} \vec{K}(\vec{r}\,') \times \left(\qquad \right) d^3\vec{r}\,'$$

1d integral

$$B(\vec{r}) = \frac{\mu_0 I}{4\pi} \int_{\text{along line}} \vec{dl} \times \left(\qquad \right)$$