PH 632

DAY 22

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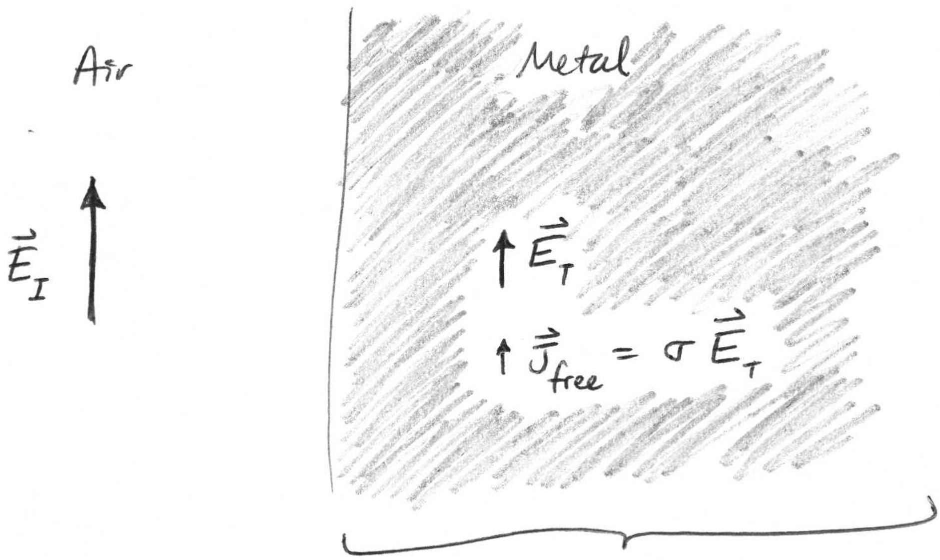


$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{— ①}$$

wave eqn inside the conducting material.

Let's look for an example of a solution $\vec{E}(\vec{r},t)$ that will satisfy the differential eqn. ①.

To make it a simple example, let $\vec{E} = E_z(x,t)\,\hat{z}$ (Note, $\nabla \cdot \vec{E} = 0$).

Then
$$\frac{\partial^2 E_z}{\partial x^2} = \mu\sigma \frac{\partial E_z}{\partial t} + \epsilon\mu \frac{\partial^2 E_z}{\partial t^2} \quad \text{— ②}$$

I'll guess the answer and see if it works

ANSATZ: $\tilde{E}_z(x,t) = E_0\, e^{i(\tilde{k}x - \omega t)}$

Note: Allow $\tilde{k}$ to be complex number.

Plug ansatz into 2 yields

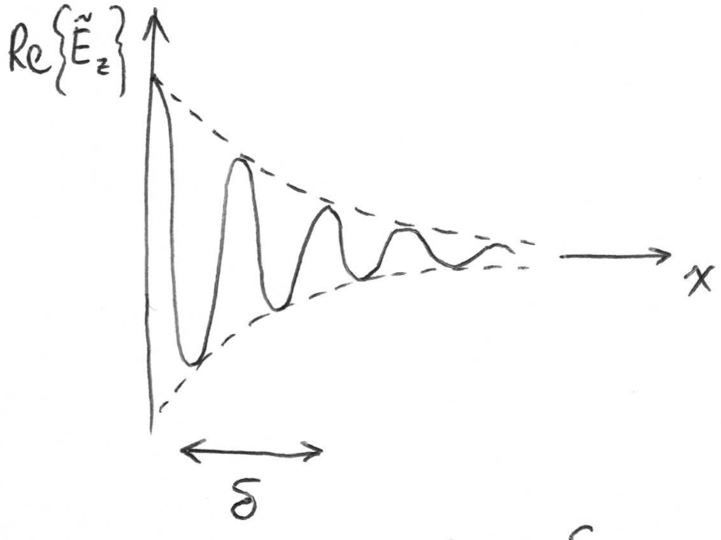
$$-\tilde{k}^2 E_z(x,t) = \mu\sigma(-i\omega)E_z(x,t) + \epsilon\mu(-\omega^2)E_z(x,t)$$

$$\tilde{k}^2 = \epsilon\mu\omega^2 + i\mu\sigma\omega$$

i.e. The wave eqn is satisfied if $\tilde{k}$ is a complex number

$$\tilde{k} = k_{Re} + i\,k_{Im}$$

and
$$\tilde{E}_z(x,t) = E_0\, e^{i(\tilde{k}x - \omega t)}$$
$$= \underset{\uparrow\ \text{exponential decay}}{E_0\, e^{-k_{Im}}}\ \underbrace{e^{i(k_{Re}x - \omega t)}}_{\text{traveling wave}}$$



Visualize the time dependence of this solution by using the PhET link on website.

The skin depth δ is defined as

$$e^{-k_{Im}\delta} = e^{-1}$$

The PhET simulation shows the wave solutions for beads on a tensioned string.

The acceleration of a bead, $\frac{d^2y}{dt^2}$,

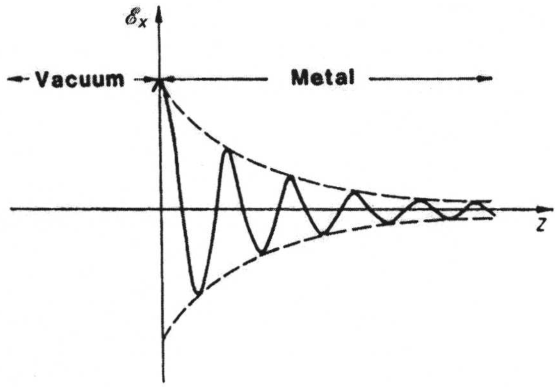
Depends on the curvature of the tensioned string $\frac{d^2y}{dx^2}$

As well as the velocity of the bead $\frac{dy}{dt}$

goes into the damping term.

Different physical system, but the differential eqn is perfectly analogous to the $\vec{E}$-field in conductor

PH632 Pop Quiz, Day 22



Find the skin depth for visible light ($\omega_{vis} \approx 10^{15}\,s^{-1}$) incident on a typical metal with $\sigma(\omega_{vis}) \approx 10^{7}\,\Omega^{-1}m^{-1}$. Assume $\varepsilon = 2\varepsilon_0$ and $\mu = \mu_0$.

I'm looking for an order of magnitude estimate, not a precise number.

Answer to pop quiz:

$$\tilde{k}^2 = \underbrace{\epsilon\mu\omega^2 + i\mu\sigma\omega}_{\text{which term is bigger?}}$$

$$= 2\epsilon_0\mu_0\omega^2 + i\mu_0\sigma\omega$$

$$= \frac{2}{c^2}(10^{15})^2 + i(4\pi\times\cancel{10^{-7}})(\cancel{10^{7}})10^{15}$$

$$= 2\times10^{13} + i\underbrace{12\times10^{15}}_{\text{almost 1000 times bigger.}}$$

$$\tilde{k} \simeq \left[10^{16} e^{i\pi/2}\right]^{1/2}$$

$$= 10^{8} e^{i\pi/4} = \frac{10^8}{\sqrt{2}}(1+i)$$

$$k_{Im} = \frac{10^8}{\sqrt{2}}$$

$$\delta = \frac{1}{k_{Im}} = \sqrt{2}\times10^{-8}\ \text{m} = 14\ \text{nm.}$$