PH632

DAY 21

Instructor:
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Finish discussion of rainbows (part (c) of last pop quiz)

See figures on website.

1. Blue light 41°
   Red light 42°

2. The arc of points that form 42° angles
   (or 41° angles).

EM WAVES IN CONDUCTORS.

Conductors are materials in which an $\vec{E}$ field causes ~~an~~ free current $\vec{J}_{free}$.

QUESTION: In intro physics, students are taught that $\vec{E}=0$ inside a conductor, so how can we say there is a non-zero $\vec{E}$ field inside a conductor?

ANSWER: $\vec{E}=0$ in a conductor is an approximation that could only be true if the conductivity, σ, of the conductor was infinite.

Most materials have a linear relationship between $\vec{J}_{free}$ & $\vec{E}$.

$$\boxed{\vec{J}_{free} = \sigma \vec{E}}$$

conductivity ... not to be confused with surface charge density.

We are seeking solutions to Maxwells eqn's inside a conductor

$$\nabla\cdot\vec{E} = 0 \qquad \nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \vec{J}_{free}$$

In a linear material

$$\nabla\times\frac{\vec{B}}{\mu} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{B} = \mu\sigma\vec{E} + \epsilon\mu\frac{\partial\vec{E}}{\partial t}$$

Now we have a pair of coupled differential eqns that link $\vec{E}$ & $\vec{B}$. (The $\nabla\times\vec{E}$ eqn and the $\nabla\times\vec{B}$ eqn).

$$\nabla\times\nabla\times\vec{E} = -\nabla^2\vec{E} = -\frac{\partial}{\partial t}\nabla\times\vec{B}$$

$$\nabla^2\vec{E} = \frac{\partial}{\partial t}\left[\mu\sigma\vec{E} + \epsilon\mu\frac{\partial\vec{E}}{\partial t}\right]$$

$$\boxed{\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \epsilon\mu\frac{\partial^2\vec{E}}{\partial t^2}}$$

Note: If $\sigma \to 0$ (i.e. an insulator)

the eqn becomes

$$\nabla^2 \vec{E} = \epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2},$$

the same wave eqn we've already studied.

What effect will the new term, $\mu\sigma \frac{\partial \vec{E}}{\partial t}$, have on the solutions to the wave eqn?

Notice that the new term is $\frac{\partial}{\partial t}$, rather than $\frac{\partial^2}{\partial t^2}$.

This is similar to how we add a damping term to the eqn of motion of a harmonic oscillator.