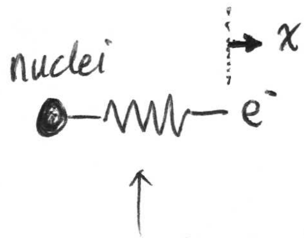
PH632 DAY 20 Instructor: Ethan Minot

THE LORENTZ-OSCILLATOR MODEL FOR REFRACTIVE INDEX

In a non conducting material, electrons are bound to atomic nuclei. As a first approximation, assume a harmonic potential holding the electrons in place.



x measures displacement from eq^b.

mechanical spring with spring constant K, resonant freq $\omega_0 = \sqrt{\frac{K}{m}}$

The electron has mass m.

$$m\ddot{x} = -eE(t) - Kx$$

time dependent $\vec{E}$-field will exert a force on the electron.

$$\ddot{x} + \frac{K}{m}x = -\frac{e}{m}E$$

$$\Rightarrow \ddot{x} + \omega_0^2 x = -\frac{e}{m}E$$

A real system will also have damping/friction. ie. a term proportional to $\dot{x}$.

$$\boxed{\ddot{x} + \gamma \dot{x} + \omega_0^2 x = -\frac{e}{m} E(t)}$$ Eqⁿ of motion.

Solve this eqⁿ of motion for $E(t) = E_0 \sin \omega t$.

It is a huge pain to solve if you insist on using real numbers. But very simple to solve if you represent $E(t)$ and $x(t)$ using complex numbers.

$$\tilde{E}(t) = E_0 e^{i\omega t}$$

Assume $\tilde{x}(t) = \tilde{x}_0 e^{i\omega t}$

allows for position to be out of phase with E-field.

Plug into eqⁿ of motion

$$\tilde{x}_0 = \frac{-e}{m} \frac{E_0}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

The dipole moment associated with the displacement of one bound electron is

$$p = -ex = \frac{e^2}{m} \frac{E_0}{\omega_0^2 - \omega^2 - i\omega\gamma} e^{i\omega t}$$

If the concentration of bound electrons is N_e, this motion produces a polarization, big P

$$P = N_e p$$

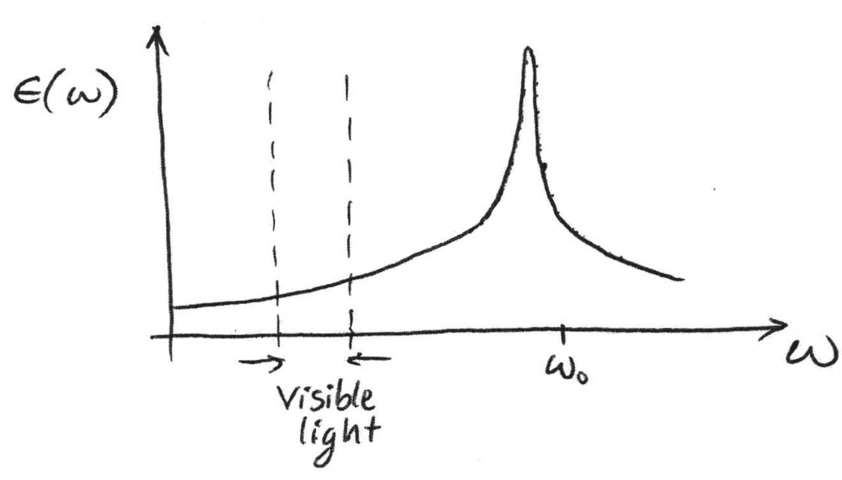
The dielectric constant of a material is defined as

$$\epsilon E = \epsilon_0 E + P$$

$$\epsilon = \epsilon_0 + \frac{P}{E}$$

$$\epsilon(\omega) = \epsilon_0 + \frac{N_e e^2}{m(\omega_0^2 - \omega^2 - i\omega\gamma)}$$

$$n = \sqrt{\frac{\epsilon}{\epsilon_0}}$$



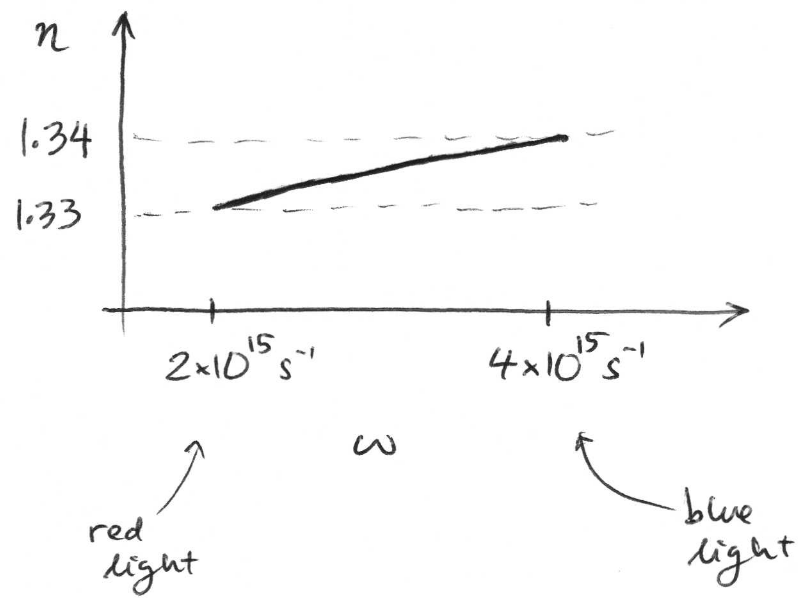
For transparent non-conducting materials

$\omega_0 > \omega_{vis}$

Small increase in ϵ accross visible spectrum.

$$n = \sqrt{\frac{\epsilon}{\epsilon_0}} = \left(1 + \frac{Ne^2}{m\epsilon_0}\,\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}\right)^{1/2}$$

Putting in realistic numbers for H_2O yields the following values for visible light in water



Now we're close to explaining rainbows.