

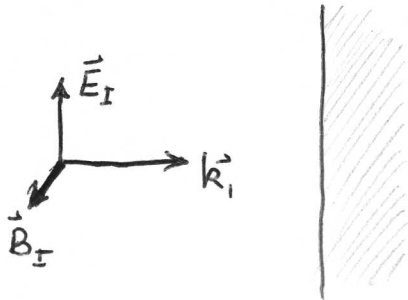
silent "s"  
Fresnel Eqns

PH632

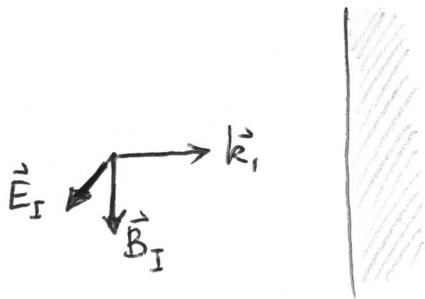
DAY 18

Calculating reflection & transmission  
of EM waves at boundaries.

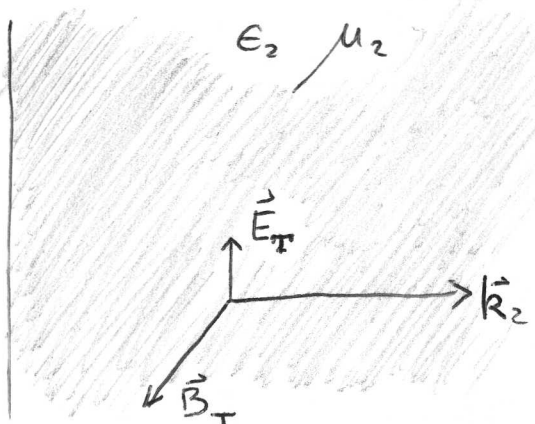
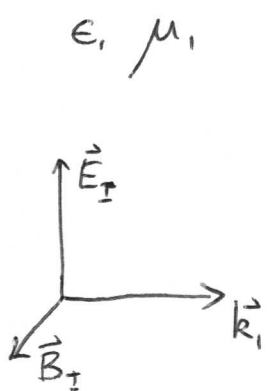
Start with normal incidence



which is equivalent to



i.e. Rotational symmetry  
about the propagation dir.  
No difference between  
s-polarized and  
p-polarized waves at normal  
incidence.



- The wavelength changes  
(so  $\vec{k}$  vector changes)
- The ratio between  $\vec{E}$  &  $\vec{B}$  changes

REFLECTED WAVE NOT SHOWN

(2)

In material #1 we have

$$\vec{E}_1 = E_{0,I} \hat{x} \sin(k_1 z - \omega t) + E_{0,R} \hat{x} \sin(-k_1 z - \omega t)$$

$$\vec{B}_1 = \sqrt{\epsilon_1 \mu_1} E_{0,I} \hat{y} \sin(k_1 z - \omega t) - \sqrt{\epsilon_1 \mu_1} E_{0,R} \hat{y} \sin(-k_1 z - \omega t)$$

Notice the negative sign.  
Since  $\vec{k}$  flips direction, the relative orientation of  $\vec{E}$  &  $\vec{B}$  also flips.

In material #2 we have

$$\vec{E}_2 = E_{0,T} \hat{x} \sin(k_2 z - \omega t)$$

$$\vec{B}_2 = \sqrt{\epsilon_2 \mu_2} E_{0,T} \hat{y} \sin(k_2 z - \omega t)$$

We want to match these fields at the boundary ( $z=0$ ).

We'll assume  $\mu_1 = \mu_2 = \mu_0$  which is common for most materials.

$$\text{Then } E_1(z=0, t) = E_2(z=0, t) \quad \text{Boundary condition 1}$$

$$B_1(z=0, t) = B_2(z=0, t) \quad \text{B.C. 2}$$

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$$\Rightarrow \left. \begin{aligned} E_{0,I} + E_{0,R} &= E_{0,T} \\ \sqrt{\epsilon_1} E_{0,I} - \sqrt{\epsilon_1} E_{0,R} &= \sqrt{\epsilon_2} E_{0,T} \end{aligned} \right\} \text{system of linear eqns.}$$

(3)

Writing in terms of refractive index

$$E_{0,I} + E_{0,R} = E_{0,T}$$

$$n_1 E_{0,I} - n_1 E_{0,R} = n_2 E_{0,T}$$

Solving this system of eqns yields

$$E_{0,R} = \left( \frac{n_1 - n_2}{n_1 + n_2} \right) E_{0,I}$$

$$E_{0,T} = \left( \frac{2n_1}{n_1 + n_2} \right) E_{0,I}$$

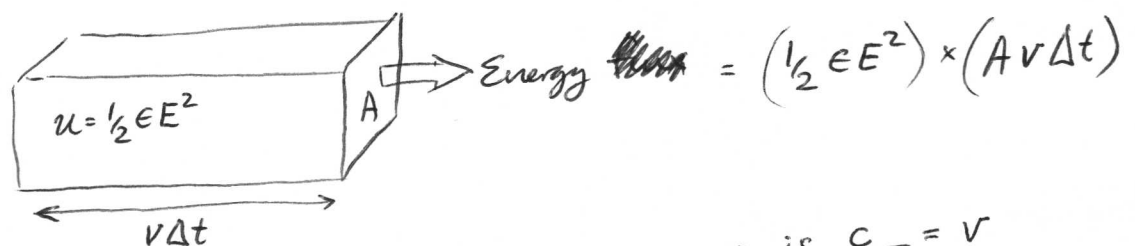
These are "Fresnel eqns" for normal incidence

$\frac{n_1 - n_2}{n_1 + n_2}$  and  $\frac{2n_1}{n_1 + n_2}$  are "Fresnel coefficients"

CONSERVATION OF ENERGY AT THE INTERFACE BETWEEN  
2 TRANSPARENT (LOSSLESS) MATERIALS.

Calculate the total energy coming into the interface  
and total energy coming out.

The energy passing through area  $A$  in time  $\Delta t$



velocity of the plane wave is  $\frac{c}{n} = v$

④

Energy flux is then

$$\frac{1}{2} \epsilon E^2 v$$

( dimensions  
 $\frac{\text{Energy}}{\text{Area} \cdot \text{Time}}$  )

This quantity is known as light intensity,  
often denoted as  $I$ . Equivalent to the magnitude  
of the Poynting vector,  $\vec{S}$ .

POP QUIZ