

## PLANE WAVES IN MATERIALS

In a material with no free current or free charge

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Assuming a linear material ( $\vec{H} = \frac{\vec{B}}{\mu}$  &  $\vec{D} = \epsilon \vec{E}$ )

we have

$$\boxed{\begin{aligned} \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} &= \mu \epsilon \frac{\partial \vec{E}}{\partial t} \end{aligned}} \quad \text{--- ①}$$

Same pair of coupled differential eqns that we had in vacuum except  $\epsilon_0 \rightarrow \epsilon$   
 $\mu_0 \rightarrow \mu$

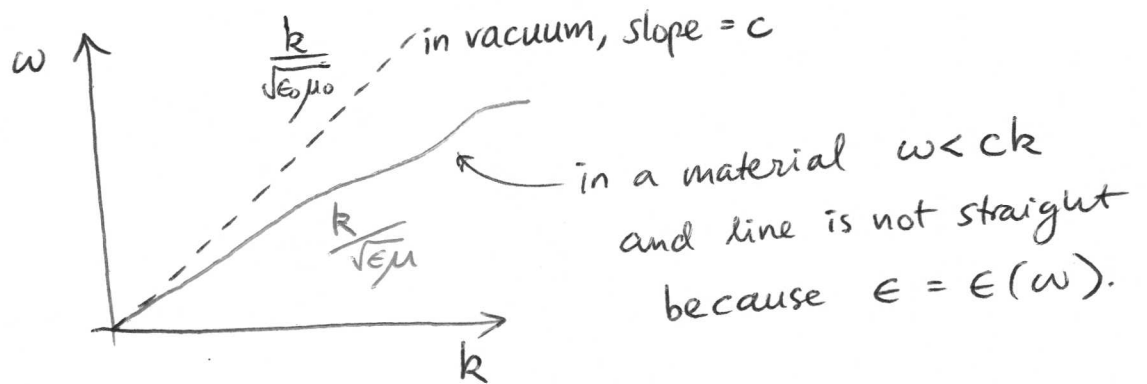
An example of a plane wave soln that satisfies ① is

$$\left. \begin{aligned} \vec{E}(z,t) &= E_0 \sin(kz - \omega t) \hat{x} \\ \vec{B}(z,t) &= \sqrt{\epsilon \mu} E_0 \sin(kz - \omega t) \hat{y} \end{aligned} \right\} \text{ where } \omega = \frac{k}{\sqrt{\epsilon \mu}}$$

Typically  $\sqrt{\epsilon \mu} > \sqrt{\epsilon_0 \mu_0}$  which means  $\vec{B}$  is a little bigger relative to  $\vec{E}$  and phase velocity is a little slower when plane wave is inside matter.

(2)

It's useful to plot the relationship between  $\omega$  &  $k$



This is called a dispersion relation ( $\omega$  vs.  $k$ ) because it allows us to calculate how much a wave packet will spread out in space (disperse) as it passes through a medium. More on that later.

Inside the material, the phase velocity is

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} = \sqrt{\frac{\epsilon_0\mu_0}{\epsilon\mu}} \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{1}{n} c$$

$n$  is called the refractive index.

$$n = \sqrt{\frac{\epsilon(\omega)\mu(\omega)}{\epsilon_0\mu_0}} \geq 1$$

Refractive index corresponds to the reduction in phase velocity.

(3)

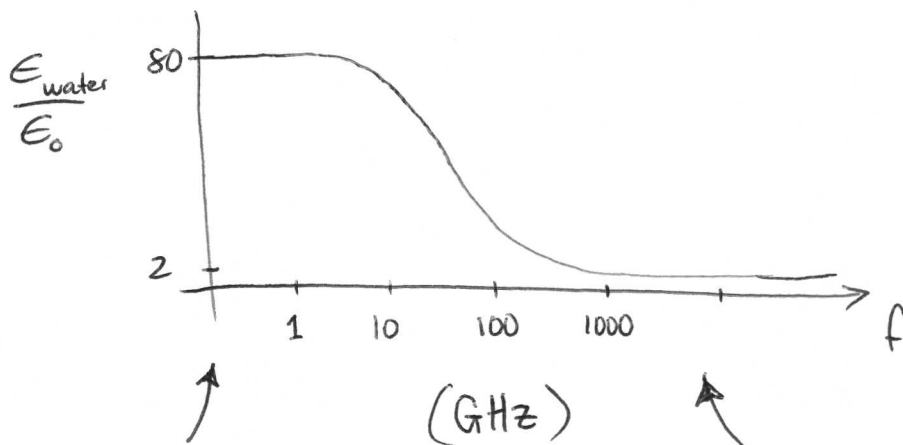
A useful rule of thumb:

$$n \approx \sqrt{\frac{\epsilon_{vis}}{\epsilon_0}}$$

where  $\epsilon_{vis}$  is dielectric constant measured at ~~visible~~ freq of visible light.

Note that  $\epsilon_{vis}$  is often very different from  $\epsilon(\omega \rightarrow 0)$

Dielectric constant of water

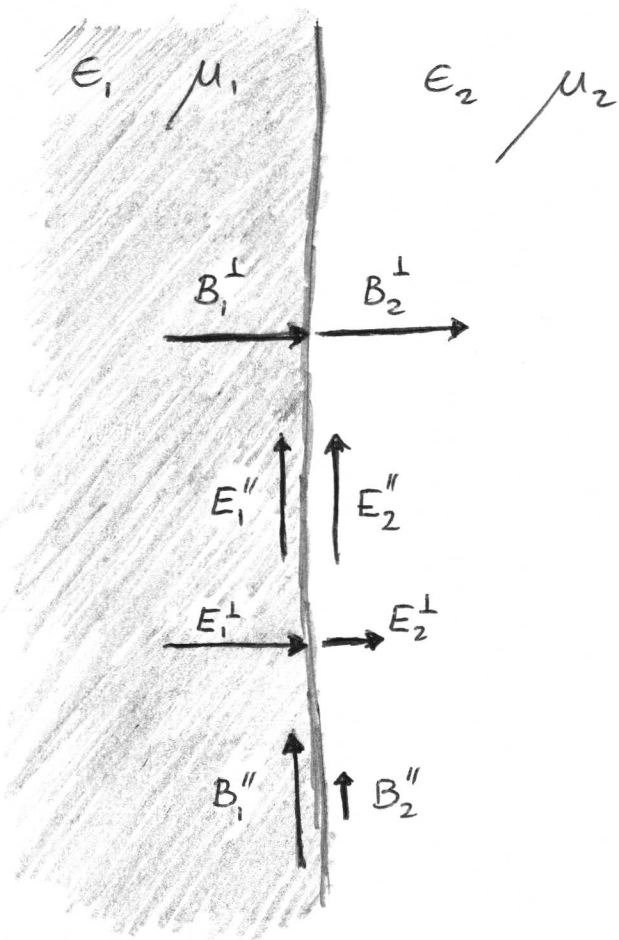


at low freqs, water ~~has time~~ molecules have time to rotate into alignment with the  $\vec{E}$  field

at high freqs, water molecules can't ~~move~~ rotate fast enough to keep up with the rapidly changing E-field.

④

# PLANE WAVES AT THE BOUNDARY BETWEEN 2 MATERIALS.



What constraints are imposed by Maxwell's eqns?

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

To answer the pop quiz:

- ① Pick the appropriate Maxwell eq<sup>n</sup>
- ② Pick the appropriate Amperian loop or Gaussian pill box