PH 632 DAY 16

PLANE WAVE POLARIZATION & PROPAGATION DIRECTION

last time
$$\vec{E}(x,y,z,t) = Re\left\{\tilde{E}_0 e^{i(kz-\omega t)}\right\}\hat{x}$$
$$\vec{B}(x,y,z,t) = Re\left\{\frac{\tilde{E}_0}{c} e^{i(kz-\omega t)}\right\}\hat{y}$$
where $\omega = ck$

Satisfies Maxwell's eqns in vacuum

This plane wave is a solution to Maxwells eqns regardless of how I set up the coord system. For example, I could relable

x-axis ⟶ z-axis

y-axis ⟶ x-axis

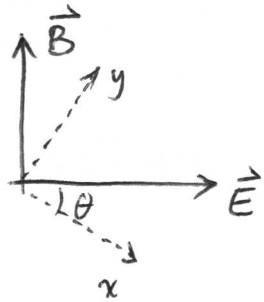
z-axis ⟶ y-axis

Then

$$\left.\begin{aligned}\vec{E}(\vec{r},t) &= Re\left\{\tilde{E}_0 e^{i(ky-\omega t)}\right\}\hat{z}\\ \vec{B}(\vec{r},t) &= Re\left\{\frac{\tilde{E}_0}{c} e^{i(ky-\omega t)}\right\}\hat{x}\end{aligned}\right]$$
satisfies Maxwell's eqns.

I want to extend this idea and generate a set of all possible plane wave solutions.

POLARIZATION



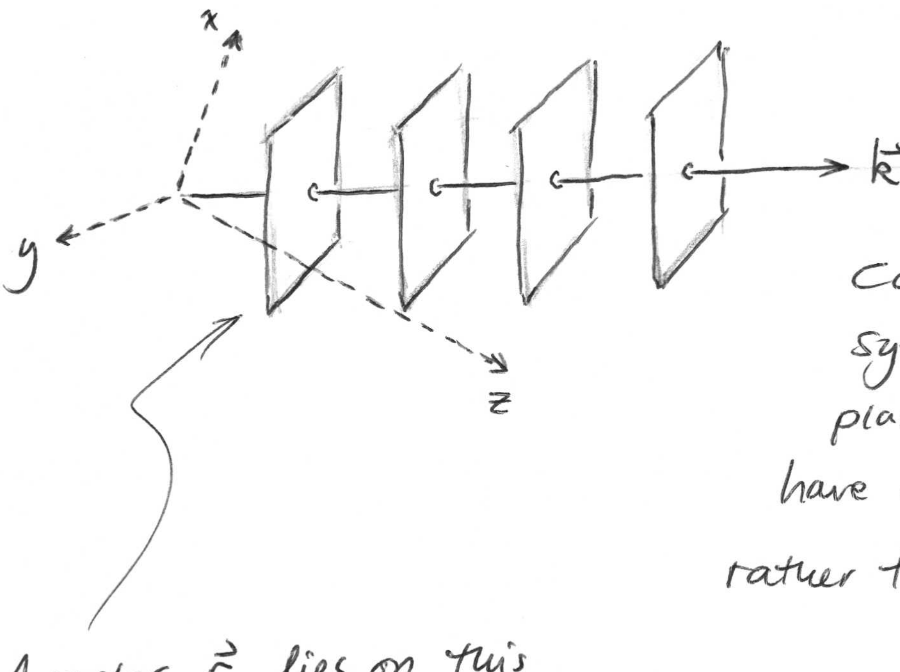
Consider a coord system where $\vec{E}$ makes an angle θ with the x-axis.

$$\vec{E} = Re\left\{ \tilde{E}_0 \hat{\epsilon}\, e^{i(kz-\omega t)} \right\}$$

$$\vec{B} = Re\left\{ \frac{\tilde{E}_0}{c}\, \hat{z}\times\hat{\epsilon}\; e^{i(kz-\omega t)} \right\}$$

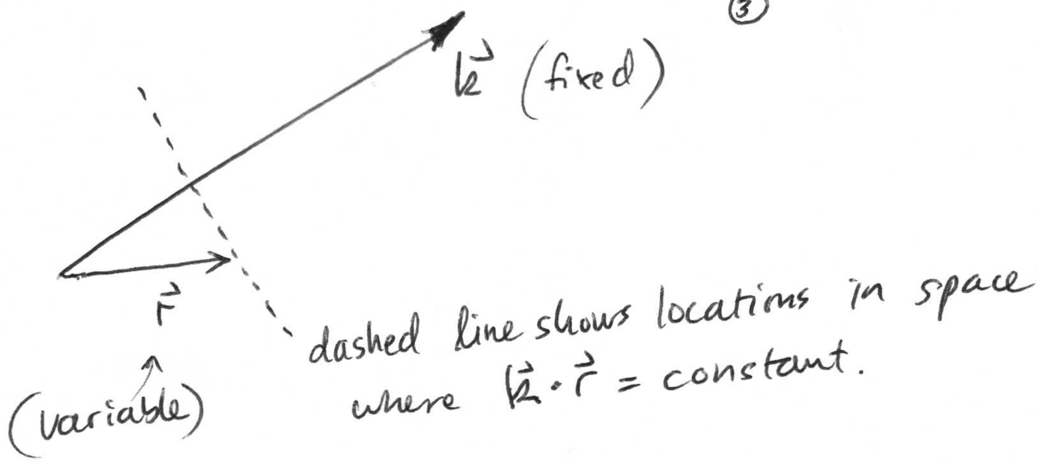
In this example, $\hat{\epsilon} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$.
$\hat{\epsilon}$ is called the polarization direction (points in direction of $\vec{E}$-field). $\vec{B}$ is always $\perp$ to $\hat{\epsilon}$.

PROPAGATION DIRECTION



Consider a coord system where the planes of constant phase have a normal vector $\vec{k}$, rather than a normal vector $\hat{z}$.

A vector $\vec{r}$ lies on this plane when $\vec{k}\cdot\vec{r}$ = constant



SUMMARY

In general, an Electromag plane wave is described by

$$\vec{E}(\vec{r},t) = \mathrm{Re}\left\{ \tilde{E}_0\, \hat{\epsilon}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \right\}$$

$$\vec{B}(\vec{r},t) = \mathrm{Re}\left\{ \frac{\tilde{E}_0 \hat{k}\times\hat{\epsilon}}{c}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \right\}$$

where $\vec{k} \perp \hat{\epsilon}$ and $\omega = ck$

Prove that $\hat{\epsilon}$ must be perpendicular to $\vec{k}$:

Proof by contradition

Assume $\vec{k} = k\hat{z}$ (as we did to generate the first plane wave soln)

Allow $\vec{E}_0$ to have a z-component so that $\vec{k}\cdot\vec{E}_0 \neq 0$.

$$\vec{E}_0 = E_{0,x}\,\hat{x} + E_{0,z}\,\hat{z}$$

Then $\tilde{\vec{E}}(\vec{r},t) = \left(E_{0,x}\,\hat{x} + E_{0,z}\,\hat{z}\right)e^{i(kz-\omega t)}$

and $\nabla\cdot\tilde{\vec{E}} = \frac{\partial}{\partial x} E_{0,x}\, e^{i(kz-\omega t)} + \frac{\partial}{\partial z} E_{0,z}\, e^{i(kz-\omega t)}$

$= 0 + ik E_{0,z}\, e^{i(kz-\omega t)}$

$$\nabla\cdot\vec{E} = \mathrm{Re}\left\{\nabla\cdot\tilde{\vec{E}}\right\} = ikE_{0,z}\; i\sin(kz-\omega t)$$

$$= -\,kE_{0,z}\sin(kz-\omega t)$$

This contradict's Maxwell's eqn $\nabla\cdot\vec{E} = 0$ in a vacuum.

Conclusion: $\vec{E}_0$ cannot have a component in the direction of propagation.