

TIME DEPENDENT MAXWELL EQNS IN MATTER

Crucial for explaining how light bounces off a raindrop (and many other phenomena)

The "bare" Maxwell's eqns are

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{E} = \rho/\epsilon_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

These work fine inside matter if we are willing to keep track of the position & motion of every elementary charged particle.

To save some effort we ~~can~~ describe the millions of electric dipoles inside a material using

$$\vec{P}$$

$$\leftarrow \frac{\text{Charge} \cdot \text{Length}}{(\text{Length})^3}$$

and describe the millions of electron orbits and spins using

$$\vec{M}$$

$$\leftarrow \frac{\text{Current} \cdot \text{Length}^2}{(\text{Length})^3}$$

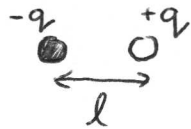
(2)

\vec{P} & \vec{M} generate \vec{E} and \vec{B} fields

From electrostatics we already know that \vec{P} generates the same \vec{E} -field as $\rho_b = -\nabla \cdot \vec{P}$

In electrodynamics we expect $\frac{\partial \vec{P}}{\partial t}$ to be analogous to a current density

Single Dipole

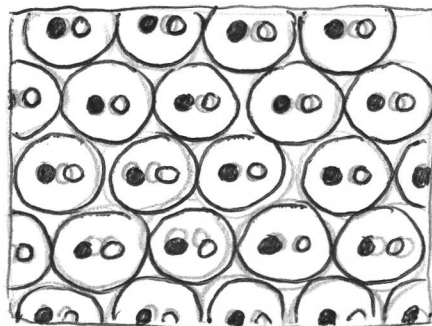


$$p = ql$$

$$\frac{dp}{dt} = q \frac{dl}{dt}$$

$$= qv$$

Solid material made from millions of identical dipoles.



$$P = nql \quad \text{where } n \text{ is concentration of dipoles}$$

$$\frac{dP}{dt} = nq \frac{dl}{dt} = nqv$$

$$\text{i.e. } \frac{d\vec{P}}{dt} = \vec{J}_P \quad \leftarrow \text{Polarization current density}$$

(3)

From magnetostatics we know that \vec{M} generates the same \vec{B} -field as $\vec{J}_b = \nabla \times \vec{M}$.

In dynamics $\frac{\partial \vec{M}}{\partial t}$ will effect \vec{E} -field because there will be non-zero $\frac{\partial \vec{B}}{\partial t}$.

In summary, we want Maxwell's eqns in matter to account for 2 types of charge density

ρ_{free} & ρ_{bound}

and account for 3 types of current density

\vec{J}_{free} , \vec{J}_{bound} & $\vec{J}_{\text{polarization}}$

① $\nabla \cdot \vec{B} = 0$ [unchanged in our new formulation]

② $\nabla \cdot \vec{E} = \frac{\rho_f + \rho_b}{\epsilon_0} = \rho_f - \frac{\nabla \cdot \vec{P}}{\epsilon_0}$

$$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\nabla \cdot \vec{D} = \rho_f \quad [\text{same as the electrostatics version}]$$

③ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ [unchanged in our new formulation. This eqn makes no reference to ρ or \vec{J}]

(4)

$$\textcircled{4} \quad \nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t} \right) + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} \right) = \vec{J}_f + \nabla \times \vec{M} + \frac{\partial}{\partial t} (\epsilon_0 \vec{E} + \vec{P})$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

[Elegantly incorporates
the effect of $\nabla \times \vec{M}$
and $\frac{\partial \vec{P}}{\partial t}$]

This term, $\frac{\partial \vec{D}}{\partial t}$, is called the displacement current density in matter.

It is the combination of $\frac{\partial \vec{P}}{\partial t}$ and $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$

A very tangible current: The motion of charge within each atom/molecule.

A more mysterious "current". It is there even in vacuum.